Reputational Spillovers

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Abstract

We analyze a reputational bargaining game in which a central player negotiates simultaneously with two peripheral players. Each player is either rational or a commitment type who never concedes and insists on a fixed share, and concessions are publicly observed. The central player's type is global, so actions in one dispute update beliefs in the other and generate reputational spillovers. The game admits a unique equilibrium, enabling a sharp comparison with the bilateral benchmark of Abreu and Gul (2000). Spillovers are payoff-relevant if and only if a peripheral is uniquely the most reputable player initially. In that case, spillovers overturn the bilateral prediction that toughness pays: the central player is never strictly better off and can be strictly worse off; the strongest peripheral loses; and the weakest peripheral can benefit, especially when the center's higher-stakes dispute is with the other peripheral.

Keywords: multilateral bargaining, reputation, spillovers.

JEL codes: C73, C78.

1. Introduction

A reputation for toughness is a strategic asset in bilateral bargaining. In the canonical reputational war of attrition of Abreu and Gul (2000), the mere possibility that one party is a commitment type who never concedes improves her terms: delay is consistent with commitment, so the opponent concedes earlier. However, many important bargaining relationships are not isolated. Governments negotiate multiple disputes in parallel, large firms bargain simultaneously with suppliers and unions, and sovereign borrowers negotiate with several creditor classes at once. When concessions are publicly observed across disputes, a concession in one negotiation can immediately affect beliefs—and hence behavior—in all others.

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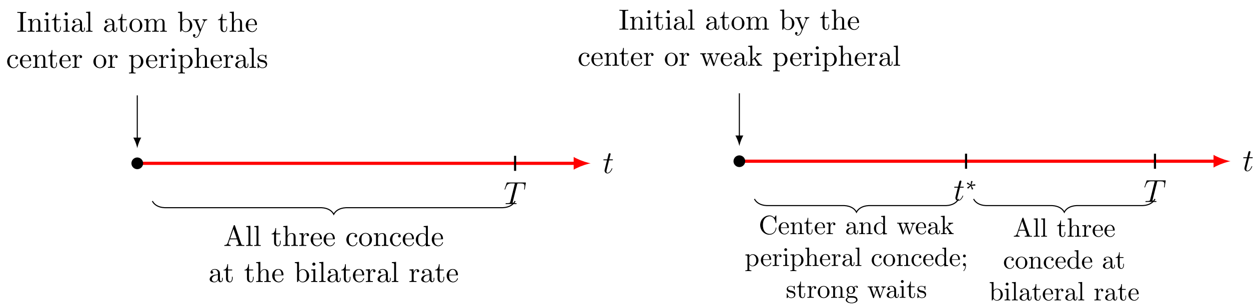
This leaves little scope for selective flexibility: yielding on one front may reveal rationality on every front. We ask whether the bilateral logic—that toughness pays—survives when a single reputation must be maintained across multiple negotiations.

To study these questions, we consider a three-player environment in which a central player bargains simultaneously with two peripheral opponents. Each bilateral negotiation follows a continuous-time war-of-attrition protocol: at any time a rational player may concede, concessions are publicly observed, and the non-conceding party receives a fixed share of the surplus. Each player is either rational or a commitment type who never concedes and insists on that share. The key feature is that the center has a *global type*: the same underlying commitment (or lack thereof) governs her behavior in both negotiations. Consequently, a concession by the center in one dispute is informative in the other as well, so beliefs about the center must remain consistent across negotiations. This cross-negotiation learning creates a belief-consistency constraint: the center cannot be perceived as tough in one dispute while behaving flexibly in the other.

Our main finding is that this constraint can overturn the bilateral prediction that toughness benefits the player who possesses it: the center's equilibrium payoff is never higher than if the two negotiations occurred in isolation, and can, in fact, be strictly lower. In contrast, a peripheral opponent who is *weaker* in the bilateral sense can be strictly better off, and the distributional consequences are systematic: the strongest peripheral never gains—and can strictly lose—from spillovers, while the weakest peripheral never loses and may strictly gain. The weakest peripheral benefits most precisely when the center's *other* negotiation is higher-stakes.

These dynamics find natural parallels in practice. Sovereign debt restructurings are a canonical example: a sovereign debtor faces multiple creditor classes simultaneously, and any concession to one class immediately reveals flexibility to the others. Our model suggests that holdout creditors—often the weakest in terms of individual exposure—can extract outsized terms precisely because the sovereign's higher-stakes negotiation lies elsewhere. The sovereign, constrained by a global reputation, cannot appear tough selectively. A similar logic arises in labor negotiations when a firm bargains simultaneously with multiple unions. If management concedes to one union, this is publicly observed and immediately undermines its bargaining position with the others—it cannot credibly claim toughness on one front after yielding on another. Our results predict that the smaller union benefits disproportionately from this constraint, especially when management's more consequential negotiation is with the larger union.

We establish these conclusions by providing a complete characterization of equilibrium behavior and payoffs. Characterizing multilateral reputational bargaining is challenging because the public state is a vector of evolving reputations, and any concession induces cross-negotiation belief updates that reshape continuation values in all unresolved disputes. As a result, equilibrium strategies typically may feature non-stationary—and potentially history-dependent—concession hazards. Despite this, we show that the simultaneous three-player environment admits a *unique* equilibrium with a sharp phase structure (Figure 1). Along the path where neither dispute has yet been resolved, behavior has the following form. There may first be a time-0 adjustment, implemented through an immediate concession atom. If one peripheral is initially sufficiently reputable, this is followed by an initial phase in which that peripheral remains inactive while the center bargains with the other peripheral. Once all players are active, all players concede at the same constant rate as in the canonical bilateral benchmark until a common finite terminal time at which remaining players are believed committed with probability one. The role of the initial phase is to reconcile cross-negotiation belief consistency with the incentives created by a global reputation.



(a) Center is most reputable

(b) A peripheral is most reputable

Figure 1: Schematic equilibrium structure.

The equilibrium structure delivers a simple and transparent condition for when spillovers are payoff relevant relative to the bilateral benchmark due to Abreu and Gul (2000) (henceforth AG).[1] Spillovers overturn the payoff rankings of AG *precisely when the player with the highest initial reputation is a peripheral.* At the beginning of the game, if the center is at least as reputable as both peripherals, the public linkage of negotiations does not distort incentives: equilibrium outcomes coincide with those from two separate bilateral reputational wars of attrition (Figure 1a). By contrast, when a peripheral begins with the strongest reputation (Figure 1b), that strong peripheral can profitably wait at the outset. This changes the center's incentives because any concession by the center is effectively *global*: it immediately resolves both disputes and reveals that the center is the flexible (rational) type. In this region, sustaining gradual concession requires a nontrivial reallocation of who concedes early and how quickly, and this reallocation drives the starkly different payoff implications relative to the AG benchmark.

The economic mechanism is intuitive. When the strongest peripheral waits initially, the center and the weaker peripheral are the only parties "bargaining in earnest" at the beginning. But the center's tradeoff in this early phase differs from the bilateral benchmark: conceding is more costly because it settles not one dispute but both, sacrificing the option value of holding out in the other negotiation. To keep the center willing to delay rather than concede immediately, the weaker peripheral must concede *faster* early on than he would in an isolated bilateral negotiation. Once all parties concede at positive rates, however, the logic

[1]As a benchmark, we compare two separate bilateral AG interactions. In the bilateral AG benchmark, the weaker player (one who is more likely to be a commitment type) concedes with an atom to the stronger player up front, and then the two players start conceding at a constant hazard rate.

of reputational bargaining pins down local incentives in a way that forces posteriors to align and concession rates to settle down to the familiar bilateral rate. The equilibrium therefore cannot absorb the early asymmetry in concession behavior smoothly over time. Instead, it reconciles early "catch-up" dynamics through *time-zero behavior*: relative to independent bilateral bargaining, immediate concessions become less concentrated on the weakest peripheral and can shift onto the center, even in the center's negotiation with the weaker opponent. This is the precise channel through which spillovers can reverse the standard bilateral intuition.

These forces generate sharp payoff implications. When the center is initially weakest, spillovers do not change the center's aggregate payoff relative to two separate bilateral negotiations, but they redistribute surplus across the peripherals: the strongest peripheral loses and the weaker peripheral gains. When the center's initial reputation lies between those of the two peripherals, spillovers are most consequential: the center's equilibrium payoff falls below the sum of her bilateral benchmark payoffs, the strongest peripheral is strictly worse off, and the weakest peripheral benefits. Moreover, the magnitude of the center's loss and the weakest peripheral's gain increases with the stakes of the center's negotiation with the stronger peripheral. Put differently, a peripheral can benefit from the center "fighting a bigger battle elsewhere," because the center's global reputation makes any concession especially costly when another high-stakes dispute is unresolved.

Furthermore, Proposition 3 shows that the payoff reversal is not driven by large commitment probabilities. Even as all three commitment probabilities vanish, the center remains strictly worse off than in the bilateral benchmark, with a payoff gap that converges to a positive limit when relative reputations are held fixed.

We also show that the disadvantage of a global reputation is not an artifact of the baseline timing, information assumptions or the number of peripherals. We study three extensions. First, we consider a sequential environment in which the center bargains with one peripheral first and only subsequently bargains with the other. In this case, the prospect of the future negotiation enters the center's incentives in the first stage and can induce asymmetric concession behavior even when initial reputations are symmetric, again potentially requiring an immediate concession by the center. Second, we relax full observability of concessions and assume that the uninvolved peripheral observes only that an agreement has been reached elsewhere (and when), but not who conceded. Then an agreement itself is informative about the center's flexibility, creating a discrete belief update that can trigger an immediate concession by the center in the remaining negotiation.

Finally, we numerically study a four-player star—a center bargaining simultaneously with three peripherals. Our simulations suggest that both the phase structure and the qualitative insight that toughness can be a liability for the central player carry over. Together, these extensions underscore that the key friction is the combination of a global type and cross-negotiation learning.

In summary, in contrast to the bilateral benchmark, where toughness pays, we show that maintaining a single reputation across multiple negotiations can reverse that logic, systematically shifting surplus away from the center and toward weaker opponents.

Our paper contributes to the literature on reputational bargaining by identifying a clean, empirically relevant environment in which reputational considerations can *harm* the player whose reputation is global. Much of the literature on reputational bargaining derives predictions for the limiting case where the probability of facing a committed opponent is small. In contrast, we treat these priors as fixed primitives and characterize equilibrium behavior and payoffs at these interior priors. This focus is motivated by evidence that delay and disagreements are empirically salient in bargaining environments. Using data from eBay's Best Offer platform, Backus et al. (2020) show that only about one-third of bargaining threads end in immediate agreement and that many negotiations end in disagreement, including after non-trivial delay. Related evidence comes from laboratory implementations of the AG protocol: Embrey et al. (2015) implement the two-stage demand-then-concession environment and find that subjects respond to the possibility of obstinate types, yet bargaining features more aggressive demands and longer conflicts than the benchmark, consistent with non-negligible "effective" obstinacy. These patterns motivate treating commitment probabilities as fixed primitives (rather than vanishingly small), so that the spillover effects we characterize are first-order; at the same time, we also study the vanishing-prior cases in Section 5 to show that reputational spillovers persist qualitatively even in the limit.

Our paper builds on the classic reputational bargaining models (Abreu and Gul (2000) and follow-up papers), where toughness pays in bilateral negotiations: players who may be "hard" types obtain better terms.[2] We depart from this benchmark by examining a multilateral environment. This connects to the network bargaining literature (Abreu and Manea, 2012; Manea, 2011), which highlights how outcomes depend on network position, and to Compte and Jehiel (2002), who show that toughness need not always pay once players face multiple opponents with outside options (see also Atakan and Ekmekci (2014)).[3] As emphasized by Fanning and Wolitzky (2022), a central difficulty in multilateral reputational bargaining is that continuation values depend on multiple reputations, so equilibrium concession rates typically evolve over time. We show that in a parsimonious multilateral environment—a central player bargaining simultaneously with two opponents—this evolution can nevertheless be characterized sharply, and it yields systematic payoff reversals relative to independent bilateral bargaining.

Our work is also related to the literature on multilateral wars of attrition. Eraslan et al. (2023) study a three-player war of attrition with majority rule and show that the central player can be worse off than in bilateral bargaining; in our setting, by contrast, inefficiency arises from reputational spillovers across simultaneous negotiations. Özyurt (2015) analyzes sequential bargaining between a buyer and two sellers, where frictions emerge from the outside option; we instead consider simultaneous bargaining, in which a single reputation must be maintained across negotiations. Kambe (2019) examines multilateral wars of attrition

[2]For follow-up papers, see for instance Abreu and Pearce (2007); Abreu and Sethi (2003); Abreu et al. (2015); Fanning (2016, 2018, 2021); Kambe (1999); Sanktjohanser (Forthcoming); Wolitzky (2012) among others. See Fanning and Wolitzky (2022) for a survey.

[3]Reversals of the "toughness pays" prediction arise through different channels in other contexts, e.g., public information about commitment (Basak, 2024) and collective bargaining with endogenous coalition structure (Ma, 2023). Our mechanism is distinct: it works through belief spillovers across simultaneously observed negotiations.

with incomplete information, where equilibrium exit patterns depend on payoff asymmetries; our contribution is to show that reputational spillovers across negotiations provide a distinct channel through which central players may be disadvantaged.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 recalls the bilateral benchmark with independent negotiations. Section 4 characterizes the unique equilibrium and derives the payoff implications and comparative statics. Section 5 studies the limit of our model as the probability of commitment types goes to zero. And finally, Section 6 studies extensions that relax simultaneity and full observability. Proofs are collected in the Appendix.

2. Model

Players and negotiations. There are three players, A, B, and C.[4] Player C is simultaneously engaged in two bilateral negotiations: one with A and one with B. Players A and B do not negotiate with each other. For $i \in \{A, B\}$, the negotiation between i and C yields a surplus $\pi_{iC} > 0$ that is realized once either party concedes in that negotiation. We normalize $\pi_{BC} = 1$.

Time and actions. Time is continuous, $t \in [0, \infty)$, and all players discount at a common rate $r > 0$. At any time t, each rational player may either wait or concede. A concession is irreversible and publicly observed. If, in the negotiation between $i \in \{A, B\}$ and C, one party concedes to the other at time t, then the party receiving the concession obtains $\alpha\pi_{iC}$ and the conceding party obtains $(1 - \alpha)\pi_{iC}$, with $\alpha > 1/2$, discounted by e^{-rt}.

Player C may, if rational, concede to A, to B, or to both; a concession to i settles the negotiation between i and C immediately. (As shown below, in equilibrium any positive-time concession by C is simultaneous across negotiations.)

Types and information. At time 0, Nature draws each player's type. Player $i \in \{A, B, C\}$ is behavioral with probability $z_i(0) \in (0, 1)$ and rational with probability $1 - z_i(0)$. Types are independent across players, and each player privately observes her own type. The prior $(z_A(0), z_B(0), z_C(0))$ is common knowledge.

A behavioral type never concedes. A rational type may concede at any time. Importantly, player C has a *single global type* that governs her behavior in both negotiations: if C is behavioral she never concedes in either negotiation; if C is rational she may concede in either negotiation.

Public histories and filtration. A *concession event* is a triple (i, j, s) indicating that player i conceded to counterparty j at calendar time s. A *public history* at time t, denoted h^t, consists of t together with the (finite) list of all concession events that occurred strictly

[4] For expositional clarity, we refer to C as "she" and to A and B as "he" throughout.

before t. Public histories h^t record all concession events that occur strictly before calendar time t. If one or more concessions occur at calendar time t, they are publicly observed immediately and may trigger further concessions with zero delay. We denote by h^{t^+} the public history immediately after all such time-t concessions have been realized. Thus t^+ refers to a post-event history node at the *same* calendar time t (no time elapses). Let $\mathcal{H}$ denote the set of public histories. Let $(\mathcal{F}_t)_{t\geq 0}$ be the filtration generated by public histories.

Strategies as stopping times. A rational player can take only one irreversible action (concede). Accordingly, a strategy for a rational player is equivalently specified by a concession time, i.e. an $(\mathcal{F}_t)$-stopping time with values in $[0,\infty]$.

For $i \in \{A, B\}$, a strategy specifies an $(\mathcal{F}_t)$-stopping time (possibly random) $\tau_i \in [0,\infty]$ at which i concedes to C. For C, we *do not impose simultaneity a priori*: a strategy specifies a pair of stopping times $(\tau_C^A, \tau_C^B) \in [0,\infty]^2$, where τ_C^A (resp. τ_C^B) is the time at which C concedes to A (resp. B). Behavioral types never concede, i.e. $\tau_i = \infty$ for $i \in \{A, B\}$ and $\tau_C^A = \tau_C^B = \infty$.

Payoffs and expected utilities. Payoffs are additive across negotiations. Fix a strategy profile σ and a belief system z. Under (σ, z), let $\tau_A, \tau_B \in [0,\infty]$ be the (possibly random) times at which A and B concede to C, and let $(\tau_C^A, \tau_C^B) \in [0,\infty]^2$ be the (possibly random) times at which C concedes to A and to B, respectively, where ∞ means "never concedes."

For each $i \in \{A, B\}$, define the resolution time of the negotiation between i and C by

$$T_{iC} := \tau_i \wedge \tau_C^i.$$

If $T_{iC} = \infty$, the negotiation between i and C is never resolved and yields payoff 0 to both parties. If both parties concede simultaneously, we assume they split the surplus equally, so each receives $\frac{1}{2}\pi_{iC}$ (discounted). If $T_{iC} < \infty$, then the realized (discounted) payoff to player i from this negotiation is

$$U_i^{iC} := e^{-rT_{iC}} \pi_{iC} \Big((1-\alpha) \mathbf{1}\{\tau_i < \tau_C^i\} + \alpha \mathbf{1}\{\tau_C^i < \tau_i\} + \tfrac{1}{2} \mathbf{1}\{\tau_i = \tau_C^i < \infty\} \Big),$$

and the realized payoff to C from the same negotiation is

$$U_C^{iC} := e^{-rT_{iC}} \pi_{iC} \Big(\alpha \mathbf{1}\{\tau_i < \tau_C^i\} + (1-\alpha) \mathbf{1}\{\tau_C^i < \tau_i\} + \tfrac{1}{2} \mathbf{1}\{\tau_i = \tau_C^i < \infty\} \Big).$$

Total realized payoffs are

$$U_A := U_A^{AC}, \qquad U_B := U_B^{BC}, \qquad U_C := U_C^{AC} + U_C^{BC}.$$

At any public history h^t, a rational type of player $k \in \{A, B, C\}$ evaluates strategies by expected utility: their continuation value is the conditional expectation

$$V_k(h^t) := E_{\sigma,z}\big[\, U_k \,\big|\, h^t \big].$$

Equilibrium. The solution concept is weak Perfect Bayesian equilibrium (PBE). A weak PBE is a pair (σ, z) consisting of a strategy profile σ and a belief system $z = (z_A, z_B, z_C)$ that assigns to each public history h^t a posterior distribution over types such that: (i) the strategy maximizes a player's expected utility given beliefs, i.e., σ is sequentially rational given z, and (ii) z is derived from σ by Bayes' rule at every history reached with positive probability under σ; off-path beliefs are unrestricted. Throughout, "equilibrium" refers to weak PBE.

Continuation games. Fix a public history h^t at which exactly one negotiation remains contested, between player $i \in \{A, B\}$ and player C. Let $z_i(h^t)$ and $z_C(h^t)$ denote the common posterior probabilities at h^t that i and C are behavioral types. Re-index time by *continuation time* $\tau := s - t \geq 0$. Then the continuation subgame from h^t is strategically equivalent (up to this change of time origin) to the bilateral AG reputational war of attrition between i and C over surplus π_{iC}, with initial reputations $(z_i(h^t), z_C(h^t))$. Since the bilateral AG game has a unique equilibrium, play in any PBE after h^t must coincide with this unique AG equilibrium. In particular, the AG equilibrium may feature an atom at continuation time $\tau = 0$, corresponding to an *immediate* concession at $t+$.

To record the relevant objects, consider a generic bilateral AG game between players i and j over surplus $\pi > 0$, with initial reputations $(z_i, z_j) \in [0, 1)^2$.[5] Let

$$g_i^j(z_i, z_j) := \max\left\{1 - \frac{z_j}{z_i},\, 0\right\}$$

denote the equilibrium probability of an *immediate concession by j to i* at continuation time $\tau = 0$ (i.e. the size of j's atom at $\tau = 0$). We sometimes just write $g_i^j(t)$ for short, when z_j and z_i are understood. Let $V_{ij}^{AG}(\pi; z_i, z_j)$ denote the equilibrium payoff to player i's *rational type* at continuation time $\tau = 0$ in this bilateral AG game. Then

$$V_{ij}^{AG}(\pi; z_i, z_j) = \pi\Big((1-\alpha) + (2\alpha - 1)\, g_i^j(z_i, z_j)\Big).$$

When π is clear from context we write $V_{ij}^{AG}(z_i, z_j)$ for short. Finally, define the *time-0 AG benchmark payoff* to player i against j by

$$v_{ij}^{AG} := V_{ij}^{AG}(\pi_{ij}; z_i(0), z_j(0)).$$

Since continuation play after any concession is uniquely determined, equilibrium behavior is fully characterized by strategies in the no-concession subgame. We therefore summarize strategies in the initial phase – when both negotiations are still contested – by concession-time distributions.

[5] We interpret expressions below by continuity at boundary cases; in particular $g_i^j(z_i, 0) = 1$ for $z_i > 0$ and $g_i^j(0, z_j) = 0$.

Strategies in the no-concession subgame. We focus on public histories at which both negotiations are still contested (i.e., no concession has occurred yet). In the no-concession subgame, the public history is summarized by calendar time t alone. We represent each player's equilibrium behavior by a cumulative distribution function (cdf) of the concession time conditional on no concession before that time. For $i \in \{A, B\}$, let $F_i : [0, \infty) \to [0, 1]$ denote the probability that i has conceded to C by time t, conditional on no prior concession. For C, let $F_C^A(t)$ (resp. $F_C^B(t)$) denote the probability that C has conceded to A (resp. B) by time t, conditional on no prior concession. These cdfs are nondecreasing and right-continuous, with $F_i(0-) = 0$.[6]

Here $F_i(t)$ is unconditional over types (but conditional on no earlier concession in the public history). Since behavioral types never concede, $F_i(t) \leq 1 - z_i(0)$ for all t.[7]

Lemma 1. *In any PBE, at any public history in which both negotiations are still contested, if C concedes to one peripheral at some calendar time $t \geq 0$, then C concedes to the other peripheral immediately as well (i.e. at calendar time $t+$). Consequently, in the no-concession subgame it is without loss to restrict attention to strategies in which $F_C^A(t) = F_C^B(t)$ for all $t \geq 0$.*

Proof. Fix a PBE (σ, z). Consider any public history at which both negotiations are still contested and at calendar time $t \geq 0$ player C concedes to A. Since behavioral types never concede, this action reveals that C is rational; hence at the post-concession history (calendar time t) the posterior belief assigns probability one to C being rational, i.e. $z_C(t+) = 0$.

From time $t+$ onward, the only remaining contested negotiation is between B and C, and the continuation game between B and C is exactly the bilateral AG war of attrition with initial reputations $(z_B(t+), z_C(t+)) = (z_B(t+), 0)$. Because $z_B(t+) \in (0, 1)$, player C has strictly lower reputation than B. In the unique AG equilibrium, the lower-reputation player concedes at continuation time 0 with probability $1 - \frac{z_C(t+)}{z_B(t+)} = 1$. Therefore, sequential rationality in the continuation subgame implies that C concedes to B immediately at calendar time $t+$.[8]

The same argument applies if C concedes to B first. Hence, whenever C concedes at a positive calendar time while both negotiations are still contested, she concedes on both negotiations immediately. This justifies representing C's no-concession strategy by a single cdf F_C for simultaneous concessions (setting $F_C(t) := F_C^A(t) = F_C^B(t)$ for $t \geq 0$). □

Because conceding in one dispute fully reveals C's type and forces immediate capitulation in the other, any concession by C is effectively a concession on both pies. Henceforth we impose $F_C^A(t) = F_C^B(t)$ for all $t \geq 0$ in the no-concession subgame and write F_C for this common cdf on $[0, \infty)$. Accordingly, for $t \geq 0$, $F_C(t)$ denotes the probability that C has

[6] Throughout, we reserve the notation $(\cdot-)$ for left limits of cdfs, e.g. $F(t-) = \lim_{s \uparrow t} F(s)$. We do not use $(\cdot+)$ to denote right limits; all cdfs are right-continuous and we write $F(0)$ for the time-0 atom.

[7] In the equilibrium characterized below, all rational concession mass is exhausted by a finite time T, so $F_i(T) = F_i(\infty) = 1 - z_i(0)$.

[8] Hence, if C concedes with positive probability to A at $t = 0$, B will not concede with positive probability to C at $t = 0$.

conceded (and therefore, by the lemma, has conceded to both A and B) by time t, conditional on no concession strictly before t.

Regularity of equilibrium strategies. We restrict attention to equilibria in which, in the no-concession subgame, each F_i may have an atom at $t = 0$ but has no atoms on $(0, \infty)$.[9] Formally, for each relevant cdf F we assume: (i) F is right-continuous and nondecreasing with $F(0-) = 0$; (ii) F is absolutely continuous on $(0, \infty)$ with density $f(\cdot) = F'(\cdot)$; (iii) f is piecewise continuous on $(0, \infty)$. Accordingly, the hazard rate $\lambda(t) := \frac{f(t)}{1-F(t)}$ is well-defined a.e. on $\{t > 0 : F(t) < 1\}$ and is piecewise continuous.

This regularity condition is imposed on the equilibrium profile being characterized; the PBE definition permits deviations that do not satisfy these regularity properties.

Posterior dynamics on the no-concession path. Along public histories with no concession up to calendar time t, let $z_i(t)$ denote the posterior probability that player i is behavioral. In the no-concession subgame, Bayes' rule implies for $t > 0$

$$z_i(t) = \frac{z_i(0)}{1 - F_i(t)} \qquad \text{whenever } F_i(t) < 1.$$

At $t = 0^+$, $z_i(0^+) = \frac{z_i(0)}{1-F_i(0)}$.

3. Benchmark: Two separate AG interactions

We recall the equilibrium outcome in the AG benchmark without reputational spillovers. In this benchmark, the two negotiations are *independent*: the history of play and the identity/timing of concessions in the negotiation between A and C are observed only by A and C (and not by B), and analogously for the negotiation between B and C. Equivalently, beliefs about C's type do not spill over across negotiations, so each negotiation is a separate bilateral reputational war of attrition à la AG with a single commitment type.

Fix a bilateral negotiation over surplus $\pi_{ij} > 0$ between players i and j, with initial reputations $z_i(0), z_j(0) \in (0, 1)$. Let $F_{ji}^{AG}(t)$ denote the (unconditional) probability that j has conceded to i by time t in the unique AG equilibrium. In each bilateral negotiation, at most one player concedes with positive probability at time 0. Player j concedes with positive probability to player i at time 0 if and only if $z_j(0) < z_i(0)$. In particular,

$$F_{ji}^{AG}(0) = \max\left\{1 - \frac{z_j(0)}{z_i(0)}, 0\right\}. \qquad \text{(Atom:AG)}$$

When $z_j(0) < z_i(0)$, we refer to j as the *weak* player and to i as the *strong* player.

[9] One can show that the PBE we characterize is the unique PBE with finitely many atoms. A proof is available on the authors' websites.

After time 0, players concede at a constant rate that makes the opponent indifferent between waiting and conceding. Specifically, player i is indifferent if

$$r(1-\alpha) = (2\alpha-1)\,\frac{f_{ji}^{AG}(t)}{1-F_{ji}^{AG}(t)},$$

where $\frac{f_{ji}^{AG}(t)}{1-F_{ji}^{AG}(t)}$ is j's hazard rate of conceding at time t. Hence, after time 0, both players concede at the constant AG hazard

$$\lambda^{AG} = \frac{r(1-\alpha)}{2\alpha-1}.$$

There is a finite terminal time T_{ij}^{AG} by which the posterior probability of being a commitment type reaches 1 and concessions stop. Let T_i^{AG} denote the time at which player i is believed committed with probability 1 (conditional on not conceding with positive probability at time 0). Then

$$T_{ij}^{AG} = \min\{T_i^{AG}, T_j^{AG}\}, \qquad T_i^{AG} = -\frac{1}{\lambda^{AG}}\log z_i(0).$$

Given this equilibrium play, player i's expected payoff in the bilateral negotiation over surplus π_{ij} is

$$v_{ij}^{AG} = \pi_{ij}\Big((1-\alpha) + (2\alpha-1)\,F_{ji}^{AG}(0)\Big).$$

Thus, in the bilateral benchmark all payoff-relevant asymmetry is absorbed by a single time-0 adjustment; after that, concession hazards are stationary at λ^{AG}.

In summary, the AG bilateral benchmark yields two key payoff properties: the stronger player receives strictly more than $(1-\alpha)\pi_{ij}$, while the weaker player receives exactly $(1-\alpha)\pi_{ij}$. We emphasize these properties because reputational spillovers in the three-player game overturn them, leading to qualitatively different concession behavior and, consequently, different payoffs.

4. Analysis and Main Result

This section characterizes equilibrium behavior and payoffs in the public, three-player bargaining game. The key friction is that player C has a *global* type, so learning about C in one dispute necessarily carries over to the other. In particular, whenever both negotiations are still unresolved, any concession by C reveals that she is rational and therefore triggers immediate resolution of *both* disputes (in equilibrium, C never concedes in only one dispute at a positive time). After the first concession, the game reduces to a standard bilateral AG reputational war of attrition in the remaining dispute.

We therefore focus on the *no-concession subgame*, i.e. the path on which neither dispute has ended yet. Along this path, each player's posterior reputation $z_i(t)$ drifts upward because not conceding is evidence of being the behavioral (commitment) type. Two basic discipline results sharply restrict equilibrium dynamics.

First, there is an *endogenous deadline*: there exists a finite time $T < \infty$ such that, conditional on no concession up to T, all remaining players are believed behavioral with probability one. Equivalently, all rational types exhaust their concession probability mass by T.

Second, before T the game cannot feature prolonged "pauses." On any interval strictly before T, at most one player can be inactive (otherwise the remaining active player can profitably shift concession mass earlier and save discounting). Moreover, once a player starts conceding with positive density, she does not stop before T (otherwise conceding right before a pause is strictly dominated by waiting an instant).

Together, these restrictions imply a sharp phase structure. There may be an initial phase in which exactly one peripheral is inactive; after at most that initial phase, all three players are active, posteriors align, and everyone concedes with the constant bilateral AG hazard until the common deadline. All departures from two independent AG negotiations are driven by a single question: *is a peripheral initially reputable enough to remain inactive at the outset?* If not, beliefs reconcile immediately at $t = 0^+$ and the public-linkage of negotiations is payoff-irrelevant. If instead a peripheral is initially dominant, equilibrium begins with a two-player phase in which that peripheral waits while C bargains with the other peripheral; because any concession by C is effectively global, this initial phase distorts early concession incentives and forces an immediate "belief reconciliation" through a time-zero concession.

A notable implication of the indifference conditions is piecewise stationarity: after at most one initial two-player phase (and a possible time-zero adjustment), concession hazards are constant and coincide with the bilateral AG hazard.

To state the equilibrium precisely, define for each player $i \in \{A, B, C\}$ the time at which i's posterior reaches one along the no-concession path,

$$T_i \;:=\; \inf\{t \geq 0 : \; z_i(t) = 1\}, \qquad \inf \varnothing = \infty.$$

We say that player i is *active* on an interval $I \subset (0, \infty)$ if F_i is strictly increasing on I (equivalently $\lambda_i(t) > 0$ for a.e. $t \in I$), and *inactive* on I if F_i is constant on I (equivalently $\lambda_i(t) = 0$ for a.e. $t \in I$). Finally, define the activation time

$$t_i \;:=\; \inf\{t > 0 : \; F_i(t) > F_i(0)\}, \qquad \inf \varnothing = \infty.$$

Proposition 1 (Equilibrium structure)**.** *There exists a unique equilibrium. Assume without loss of generality that $z_A(0) \geq z_B(0)$. Along the no-concession path:*

(i) There exists $T < \infty$ such that $T_i = T$ for all $i \in \{A, B, C\}$.

(ii) If $z_A(0) > \max\{z_B(0), z_C(0)\}$, then $t_A \in (0, T)$ and $t_B = t_C = 0$ with

$$\lambda_B(t) = (1 + \pi_{AC})\lambda^{AG}, \qquad \lambda_C(t) = \lambda^{AG} \quad \textit{for a.e. } t \in (0, t_A).$$

The identity and size of the time-0 atom (at most one of B and C places mass at 0)

are uniquely pinned down by the posterior-alignment condition at the activation time:

$$z_B(t_A) = z_C(t_A) = z_A(0).$$

For a.e. $t \in (t_A, T)$*, all three players concede with hazard* λ^{AG}*.*

(iii) If $z_A(0) \leq \max\{z_B(0), z_C(0)\}$*,* $t_i = 0$ *for all* $i \in \{A, B, C\}$*. Any time-*0 *atoms are uniquely pinned down by immediate posterior alignment:*

$$z_A(0^+) = z_B(0^+) = z_C(0^+),$$

and for a.e. $t \in (0, T)$ *all three players concede with hazard* λ^{AG}*.*

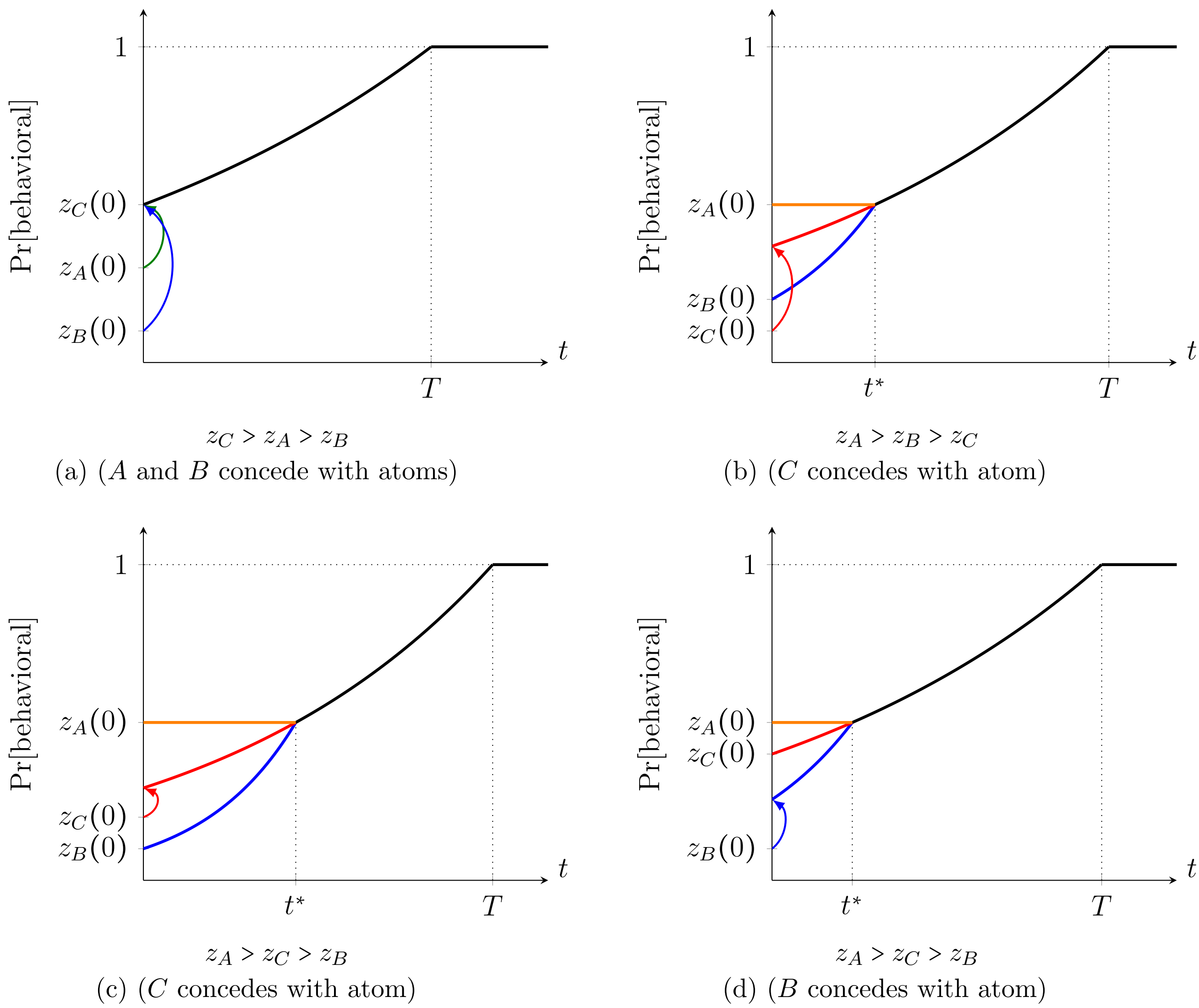


Figure 2: Equilibrium posterior dynamics under four configurations.

Proposition 1 leaves one object to be determined: when a peripheral is strictly dominant, which player concedes at time 0, and with what probability? We now show that the answer follows from the requirement of posterior alignment at the moment the last player becomes active. A complete version of Proposition 1 is presented in Proposition A.1 in the appendix.

Suppose $z_A(0) > \max\{z_B(0), z_C(0)\}$, so that A is initially inactive while B and C are active on $(0, t_A)$. On that interval, Proposition 1 implies constant hazards

$$\lambda_C = \lambda^{AG}, \qquad \lambda_B = (1 + \pi_{AC})\lambda^{AG}.$$

Hence along the no-concession path,

$$z_C(t) = z_C(0^+)e^{\lambda^{AG}t}, \qquad z_B(t) = z_B(0^+)e^{(1+\pi_{AC})\lambda^{AG}t}, \qquad z_A(t) = z_A(0).$$

Because joint activity forces posterior alignment, the activation time t_A must satisfy

$$z_B(t_A) = z_C(t_A) = z_A(0). \tag{Alignment}$$

First consider the hypothetical case with no time-zero atoms: $z_i(0^+) = z_i(0)$ for $i \in \{B, C\}$. Define the catch-up times

$$\tilde{t}_C = \frac{1}{\lambda^{AG}} \log\left(\frac{z_A(0)}{z_C(0)}\right), \qquad \tilde{t}_B = \frac{1}{(1 + \pi_{AC})\lambda^{AG}} \log\left(\frac{z_A(0)}{z_B(0)}\right).$$

These are the times at which C and B, respectively, would reach $z_A(0)$ under the initial-phase hazards. In other words, these times compare the endogenous growth speeds of reputations under the initial-phase hazard rates. If $\tilde{t}_C = \tilde{t}_B$, no time-zero atom is required: both posteriors reach $z_A(0)$ simultaneously. If instead $\tilde{t}_C > \tilde{t}_B$, then absent atoms C would reach $z_A(0)$ strictly later than B. To satisfy the alignment condition, C must therefore receive an upward jump in her posterior at time 0, implemented by a time-zero concession atom. Conversely, if $\tilde{t}_B > \tilde{t}_C$, then B is the laggard and must concede with positive probability at time 0.

Consider the case $\tilde{t}_C > \tilde{t}_B$, so that C must concede at 0. Since C places mass $F_C(0)$ at time 0, her posterior jumps to

$$z_C(0^+) = \frac{z_C(0)}{1 - F_C(0)}.$$

The activation time is then determined by B's catch-up time,

$$t_A = \frac{1}{(1 + \pi_{AC})\lambda^{AG}} \log\left(\frac{z_A(0)}{z_B(0)}\right).$$

Imposing alignment $z_C(t_A) = z_A(0)$ yields

$$z_C(0^+) = z_A(0)\left(\frac{z_B(0)}{z_A(0)}\right)^{\frac{1}{1+\pi_{AC}}} = z_A(0)^{\frac{\pi_{AC}}{1+\pi_{AC}}} z_B(0)^{\frac{1}{1+\pi_{AC}}}.$$

Hence

$$F_C(0) = 1 - \frac{z_C(0)}{z_A(0)^{\frac{\pi_{AC}}{1+\pi_{AC}}} z_B(0)^{\frac{1}{1+\pi_{AC}}}}.$$

An analogous calculation yields the formula in the case where B is the laggard. The time-zero atom is therefore assigned to whichever active player would otherwise "arrive late" to $z_A(0)$ under the initial hazard rates. Because B concedes at the accelerated rate $(1 + \pi_{AC})\lambda^{AG}$, increasing π_{AC} reduces $\tilde{t}_B$ and makes it more likely that C is the laggard. Thus sufficiently high stakes in the A–C negotiation shift the time-zero concession onto the center.

Let t^* denote the time at which posteriors first align and all three players are active (so $t^* = t_A$ in Proposition 1(ii), and $t^* = 0$ in Proposition 1(iii). Figure 2 illustrates the posterior paths described in Proposition 1. Each panel plots posterior commitment probabilities $z_i(t)$ along the no-concession path. Vertical jumps at $t = 0$ correspond to time-0 concession atoms, which raise the non-conceding player's posterior to $z_i(0+) = z_i(0)/(1 - F_i(0))$. In panels (b)-(d), A is initially inactive, while B and C are active with constant hazard rates $\lambda_B = (1+\pi_{AC})\lambda^{AG}$ and $\lambda_C = \lambda^{AG}$, so z_B grows faster than z_C until the activation time t^* when posteriors align at $z_A(0)$. Panels (c) and (d) differ by which active player is the laggard: if $\tilde{t}_C > \tilde{t}_B$, then C bears the time-0 atom; if $\tilde{t}_B > \tilde{t}_C$, then B bears the time-0 atom. After alignment at t^*, all players concede at hazard λ^{AG} until the common terminal time T.

Proposition 1 pins down the equilibrium path up to a single remaining degree of freedom: the identity and size of any time–0 concession atom. This atom is needed to reconcile the (initially asymmetric) posterior growth rates in the initial phase with the requirement that posteriors coincide when the last player becomes active. As discussed above, the posterior-alignment condition at the activation time uniquely selects this atom.

Once this object is fixed, equilibrium behavior is fully characterized. After at most an initial two-player phase, all active players concede at the bilateral AG hazard λ^{AG} until a common terminal time. This phase structure makes transparent where equilibrium payoffs can differ from the benchmark of two independent bilateral Abreu and Gul (2000) negotiations: since the no-concession dynamics coincide with λ^{AG} after the initial adjustment, any payoff wedge must be generated entirely by that initial adjustment—most notably by the time–0 atom required for posterior alignment when a peripheral is initially dominant. We now formalize this comparison.

Before stating Proposition 2, let us define expected payoffs. Fix a strategy profile σ. For each player $i \in \{A, B, C\}$, let $v_i(\sigma)$ denote the *time*-0 expected payoff of i's *rational type* under σ. We write $v_i^* := v_i(\sigma^*)$ for the equilibrium payoff. To connect payoffs to the indifference conditions used in the Appendix, it is convenient to define the payoff to a peripheral from conceding at a deterministic time. Fix $i \in \{A, B\}$, let k denote the other peripheral, and consider the plan:

> "If no one concedes strictly before t, concede to C at time t. If k concedes at some $y < t$ before C concedes, then from calendar time y onward play the unique AG equilibrium in the remaining i–C negotiation."

Let $U_i(t;\sigma_{-i})$ be the expected payoff at time 0 from this plan, and write $F_m(t-) := \lim_{s\uparrow t} F_m(s)$ for left limits. Then a first-event decomposition yields

$$U_i(t;\sigma_{-i}) = \alpha\pi_{iC}\int_{[0,t)} e^{-ry}\big(1-F_k(y-)\big)\,dF_C(y) \;+\; \int_{[0,t)} e^{-ry}\big(1-F_C(y)\big)\,V_{iC}^{AG}\big(z_i(y),z_C(y)\big)dF_k(y)$$
$$+\;(1-\alpha)\pi_{iC}e^{-rt}\big(1-F_C(t)\big)\big(1-F_k(t-)\big), \qquad (1)$$

where we recall that $V_{iC}^{AG}(z_i,z_C)$ denotes i's continuation payoff in the induced bilateral AG game starting at calendar time y when the remaining negotiation starts with reputations (z_i,z_C).[10] The three terms in (1) correspond, respectively, to: (i) C concedes before t and before k concedes; (ii) k concedes before t and before C concedes, inducing the AG continuation in i–C; and (iii) no one concedes before t, so i concedes at t.[11]

In the equilibrium characterized below, once a peripheral becomes active he is indifferent over the times in the support of his concession distribution, so his equilibrium payoff, v_i^*, can be recovered from (1) by evaluating $U_i(\cdot;\sigma_{-i}^*)$ at any such time (e.g. at $t_i := \inf\{t\ge 0 : F_i^*(t)>0\}$).

An analogous decomposition applies to C.[12] Define $U_C(t;\sigma_{-C})$ as the time-0 payoff to C's rational type from the plan:

> "Do not concede before t; if no one concedes before t, concede to both A and B at time t; if A (resp. B) concedes first at some $y<t$, then from y onward play the unique AG equilibrium in the remaining B–C (resp. A–C) negotiation."

Recall that $V_{CB}^{AG}(z_C,z_B)$ (resp. $V_{CA}^{AG}(z_C,z_A)$) denotes C's continuation payoff in the induced AG game, i.e., the remaining B–C (resp. A–C) negotiation when it starts with reputations (z_C,z_B) (resp. (z_C,z_A)). Then

$$\begin{aligned}U_C(t;\sigma_{-C}) = {}&\alpha(\pi_{AC}+1)\,F_A(0)F_B(0)\\ &+\int_{[0,t)} e^{-ry}\big(1-F_B(y)\big)\Big(\alpha\pi_{AC}+V_{CB}^{AG}\big(z_C(y),z_B(y)\big)\Big)dF_A(y)\\ &+\int_{[0,t)} e^{-ry}\big(1-F_A(y)\big)\Big(\alpha+V_{CA}^{AG}\big(z_C(y),z_A(y)\big)\Big)dF_B(y)\\ &+(1-\alpha)(\pi_{AC}+1)\,e^{-rt}\big(1-F_A(t)\big)\big(1-F_B(t)\big).\end{aligned} \qquad (2)$$

The time-0 concession events are partitioned as follows. The explicit term $F_A(0)F_B(0)$ isolates the event that both peripherals concede at 0. Because the Lebesgue–Stieltjes integrals are taken over $[0,t)$, they also pick up any atoms at $y=0$; however, the factor $1-F_B(y)$ (resp. $1-F_A(y)$) implies that the atom of dF_A at 0 (resp. dF_B at 0) contributes only on the one-sided event that A concedes at 0 while B does not (resp. that B concedes at 0 while A

[10] All integrals are Lebesgue–Stieltjes, so they incorporate any atoms of F_C and F_k at time $t=0$.

[11] Under the regularity restriction we impose on the *equilibrium profile*, there is no simultaneous-concession term for $t>0$.

[12] Recall that by Lemma 1, it is without loss on the equilibrium path to treat C's concession in the no-concession subgame as simultaneous across negotiations.

does not).[13]

We can now state Proposition 2 which compares each player's equilibrium expected payoff under public negotiations—where reputational spillovers are present—with the benchmark of two independent bilateral negotiations à la AG from Section 3.

Proposition 2 (Equilibrium payoffs)**.** *Assume without loss of generality that* $z_A(0) \geq z_B(0)$*. In the unique equilibrium:*

1. *If* $z_A(0) \leq \max\{z_B(0), z_C(0)\}$*, then*

$$v_i^* = v_{i,C}^{AG} \text{ for } i \in \{A, B\}, \qquad v_C^* = v_{C,A}^{AG} + v_{C,B}^{AG}.$$

2. *If* $z_A(0) > z_B(0) \geq z_C(0)$*, then*

$$v_A^* < v_{A,C}^{AG}, \qquad v_B^* > v_{B,C}^{AG}, \qquad v_C^* = v_{C,A}^{AG} + v_{C,B}^{AG}.$$

3. *If* $z_A(0) > z_C(0) > z_B(0)$*, then*

$$v_A^* < v_{A,C}^{AG}, \qquad v_B^* \geq v_{B,C}^{AG}, \qquad v_C^* < v_{C,A}^{AG} + v_{C,B}^{AG}.$$

Proposition 2 is the payoff counterpart of the phase structure in Proposition 1. When $z_A(0) \leq \max\{z_B(0), z_C(0)\}$, no peripheral is *uniquely* most reputable. In this region Proposition 1(iii) implies immediate posterior alignment at 0^+, and from that point onward the no-concession dynamics coincide with the bilateral AG hazard λ^{AG}. Hence reputational spillovers are payoff-irrelevant: each player's expected payoff coincides with the bilateral AG benchmark in each dispute.

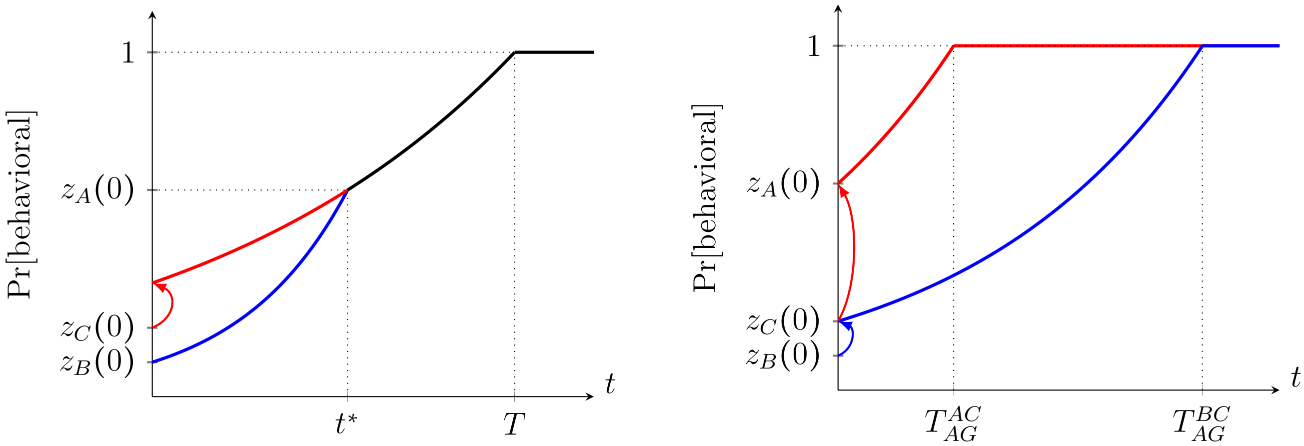
Spillovers are payoff-relevant if and only if a peripheral is initially dominant, $z_A(0) > \max\{z_B(0), z_C(0)\}$. Then Proposition 1(ii) implies an initial two-player phase in which the dominant peripheral A optimally remains inactive while B and C "bargain in earnest." Because C's type is global, any concession by C is effectively *global*—it resolves both negotiations at once and reveals C's flexibility. This makes conceding comparatively more costly for C in the initial phase and forces an initial reallocation of concession probability (including, when required by posterior alignment, a time–0 atom pinned down by the condition that posteriors coincide when A becomes active).

The payoff implications are then systematic and mirror the two subregions in the dominant-peripheral case. If $z_A(0) > z_B(0) \geq z_C(0)$, spillovers operate purely through redistribution across the peripherals: the dominant peripheral A is strictly worse off than in the bilateral benchmark, while the weaker peripheral B is strictly better off, and the center's aggregate payoff remains exactly at the sum of her bilateral AG payoffs. If instead $z_A(0) > z_C(0) > z_B(0)$, spillovers are most consequential: the center is *strictly* worse off than in the bilateral benchmark, the dominant peripheral again loses, and the weakest peripheral

[13] Thus simultaneous time-0 concessions are counted only in the first term.

weakly benefits. In particular, the bilateral "toughness pays" logic is overturned precisely in the region where belief consistency forces a nontrivial initial adjustment.

Figure 3 provides a numerical illustration of Proposition 2 by contrasting the equilibrium posterior dynamics with those under two independent AG negotiations. The figure makes clear that the payoff wedge is generated entirely by the initial adjustment (including any time–0 atom) required for posterior alignment.



(a) Multilateral bargaining

(b) Bilateral bargaining (AG benchmark)

Figure 3: Comparison of concession behavior: multilateral bargaining (left) vs. bilateral bargaining (right).

Specifically, Figure 3 illustrates the equilibrium concession dynamics with reputational spillovers and contrasts it to the bilateral AG benchmark. In the multilateral game (left panel), player C (in red) concedes with positive probability to both A and B at $t = 0$, producing an upward jump in C's posterior reputation at time 0 (to $z_C(0+)$) to maintain consistent posterior beliefs across the two negotiations. Thereafter, the initial two-player phase begins: B concedes at a hazard rate of $2\lambda^{AG}$ (in blue) and C concedes at a rate of λ^{AG}. Given these rates of concession and the initial atom by C, they reach A's prior $z_A(0)$ at time $t^* = t_A$. Once A's prior is reached, all three players concede at a rate λ^{AG} up to time T, where all three players are believed to be committed with probability 1.

The right panel considers the corresponding bilateral benchmarks. In the bilateral negotiation between A and C, C is initially the weaker player ($z_C(0) < z_A(0)$) and therefore concedes with an atom to A at time 0, raising the common posterior to $z_A(0)$. Thereafter, both players concede at a rate λ^{AG} until T_{AC}^{AG}. In the bilateral negotiation between B and C, B is initially the weaker player ($z_B(0) < z_C(0)$) and therefore concedes with an atom to C at time 0, raising the common posterior to $z_C(0)$. Thereafter, both players concede at a rate λ^{AG} until T_{BC}^{AG}.

Two contrasts with the multilateral equilibrium are immediate. First, in the bilateral benchmark the weaker player in each negotiation is the one who concedes with positive probability at time 0 (here C in the negotiation between A and C and B in the negotiation between B and C), whereas in the multilateral environment C concedes with an atom to both players even though $z_C(0) > z_B(0)$. Second, the multilateral equilibrium exhibits a longer common terminal time than the bilateral benchmark between A and C ($T > T_{AC}^{AG}$) but a shorter terminal time than the bilateral benchmark between B and C ($T < T_{BC}^{AG}$), reflecting the accelerated concession rate of the weak peripheral during the initial two–player phase.

Propositions 1 and 2 imply that any departure from the benchmark of two independent AG negotiations is generated entirely by the initial adjustment required for posterior alignment—after alignment, concession hazards coincide with the bilateral AG rate and the subsequent dynamics are identical. In the dominant-peripheral region, this adjustment takes the form of a single time-0 atom borne by the player who would otherwise 'arrive late' to the dominant peripheral's reputation under the initial-phase hazards. The closed-form atom pinned down by the alignment condition therefore delivers immediate comparative statics in the stakes parameter π_{AC}. If B is the bearer of the atom, C's "reputation rent" from B's immediate concession shrinks as π_{AC} increases. If C is the bearer of the atom, the spillover component of the peripherals' equilibrium payoffs is increasing in π_{AC}: a higher π_{AC} raises the probability that C concedes immediately at time 0, which benefits both peripherals.

5. A vanishing-prior limit with fixed relative reputations

A natural question is whether the payoff reversal in Proposition 2 is an artifact of treating commitment probabilities as fixed primitives or whether it survives when those probabilities become small. To address this, we study a vanishing-prior limit along sequences for which all three priors converge to zero while relative reputations remain fixed.

Throughout this section, we set $\pi_{AC} = \pi_{BC} = 1$ and focus on the region $z_A(0) > z_C(0) > z_B(0)$, where spillovers are most consequential (Proposition 2, case 3). Fix constants $\kappa_A, \kappa_B > 1$ and, for each sufficiently small $\varepsilon > 0$, define

$$z_B^\varepsilon(0) = \varepsilon, \qquad z_C^\varepsilon(0) = \kappa_B \varepsilon, \qquad z_A^\varepsilon(0) = \kappa_A \kappa_B \varepsilon.$$

Thus,

$$\frac{z_A^\varepsilon(0)}{z_C^\varepsilon(0)} = \kappa_A, \qquad \frac{z_C^\varepsilon(0)}{z_B^\varepsilon(0)} = \kappa_B$$

for every ε, while all three commitment probabilities vanish as $\varepsilon \downarrow 0$. Let $v_i^*(\varepsilon)$ denote player i's equilibrium payoff in the multilateral game under these priors, and let $v_{ij}^{AG}(\varepsilon)$ denote player i's payoff in the corresponding bilateral AG benchmark against j.

Proposition 3 (Vanishing-prior limit). *As $\varepsilon \downarrow 0$,*

$$v_{CA}^{AG}(\varepsilon) + v_{CB}^{AG}(\varepsilon) - v_C^*(\varepsilon) \longrightarrow (2\alpha - 1)\frac{\min\{\kappa_A, \kappa_B\} - 1}{\kappa_B} > 0,$$
$$v_{AC}^{AG}(\varepsilon) > v_A^*(\varepsilon) \qquad \text{for every } \varepsilon > 0,$$
$$v_B^*(\varepsilon) \geq v_{BC}^{AG}(\varepsilon) \qquad \text{for every } \varepsilon > 0,$$

with strict inequality in the last display if and only if $\kappa_A > \kappa_B$.

The limit in Proposition 3 is directional: it depends on the fixed ratio pair (κ_A, κ_B). When $\kappa_A > \kappa_B$, player C is the laggard under the initial-phase hazards and therefore bears the time-zero atom, which benefits both peripherals. When $\kappa_A \leq \kappa_B$, player B is the laggard and concedes at time zero instead. Even in that case, the center remains strictly worse off than under two independent bilateral AG negotiations, because the time-zero adjustment is smaller than the bilateral concession that B would make to C in isolation.

6. Discussion and robustness

This section discusses some extensions that relax key modeling assumptions of the baseline such as the the simultaneity of negotiations, full observability of concessions, or that there are only two peripherals, and illustrates how the mechanism identified above operates in these environments.

Sequential negotiations: A maintained assumption of the baseline model is that both negotiations are simultaneous. In Section B of the Online Appendix, we study a sequential environment in which A and C bargain first, and the negotiation between B and C begins only once the first dispute is resolved. Proposition B.1 shows that the equilibrium characterization carries over: there is a unique equilibrium with a common terminal time and piecewise-constant hazard rates, and the time-0 atom (if any) is uniquely pinned down by a boundary condition on posteriors. The key finding is that the central player's disadvantage is not an artifact of simultaneity. In fact, the sequential environment sharpens it: even when all three players have *identical* initial reputations, player C may be forced to concede with a positive atom at time 0, and is strictly worse off than in the bilateral benchmark. The reason is that the prospect of a future negotiation with B enters C's indifference condition in the first stage. This raises the effective cost to C of holding firm against A. This is because conceding to A is now less costly since it triggers the second-stage game in which C retains her reputation. As a result, A concedes at a strictly higher rate than λ^{AG}, his posterior grows faster than C's, and belief alignment at the terminal time can require an upfront concession by C even when no asymmetry in initial reputations would have compelled one in the bilateral benchmark.

Partial observability. A second assumption we relax is full observability of concessions. In Section C of the Online Appendix, we study the case in which the uninvolved peripheral observes only the timing of an agreement, not the identity of the conceding party. An agreement is then informative about C's type regardless of who conceded: since commitment types never concede, observing a settlement causes the uninvolved peripheral to revise downward his belief that C is behavioral. This downward jump can make C the perceived weaker party in the remaining negotiation and force an immediate concession by C the instant the outside agreement is observed. Proposition B.2 shows that this force operates even when all three players have identical initial reputations—a case where the baseline features no atoms and all players concedesymmetrically at λ^{AG}. Under partial observability, C is forced to concede with a strictly positive atom at $t = 0$ and at a rate strictly below λ^{AG} thereafter, while A and B continue to concede at λ^{AG} and earn strictly more than in the bilateral benchmark. As a consequence, even if C is the strongest player initially, she may concede with an atom at time 0 yielding a strictly lower payoff than the bilateral benchmark. Thus, partial observability amplifies rather than attenuates the central player's disadvantage.

Beyond the three-player star: A natural question is whether the mechanism identified in the three-player star extends to richer star networks. Section D in the Online Appendix derives the corresponding local indifference conditions for an equal-pie four-player star. Relative to the three-player case, these conditions depend on continuation values from the three-player equilibrium, so the implied hazard system is a system of ODEs leading to time-varying hazard rates.

Section D also reports a numerical solution for a representative parameterization. In that example, the equilibrium exhibits a sequential activation pattern and preserves the same qualitative force as in the three-player model: the weakest peripheral benefits from spillovers, while the center is worse off relative to the bilateral benchmark. More broadly, the structure of the four-player system suggests that the methods developed here can be extended to obtain a fuller analytical characterization of the n-peripheral star.

Together, these extensions reinforce the main message of the paper. Reputational spillovers arise from the combination of a global type and belief consistency across negotiations. Relaxing simultaneity, full observability, or adding more peripheral players does not eliminate these forces and can continue to make the central player worse off relative to the bilateral benchmark.

A. Appendix: Proofs

A.1. Notation and Preliminaries.

Throughout the appendix we analyze the *no-concession subgame*, i.e., the subgame in which both negotiations between A and C and between B and C are still contested. In this subgame the public history is summarized by calendar time $t \geq 0$.

Concession-time distributions. Fix an equilibrium profile in the no-concession subgame and let $F = (F_A, F_B, F_C)$ denote the induced cdfs of concession times. For each player $i \in N := \{A, B, C\}$, $F_i(t)$ is the probability (under the equilibrium profile) that player i has conceded by time t *conditional on no earlier concession*. Since behavioral types never concede, $F_i(t) \le 1 - z_i(0)$ for all t, and in the equilibrium characterized below rational types concede with probability one along the no-concession path, so $F_i(\infty) = 1 - z_i(0)$.

For $t > 0$ write $F_i(t-) := \lim_{s \uparrow t} F_i(s)$.

Posteriors, hazard rates, and "survival". Let $z_i(t)$ denote the posterior probability that i is behavioral conditional on no concession up to time t. Bayes' rule implies that whenever $F_i(t) < 1$,

$$z_i(t) = \frac{z_i(0)}{1 - F_i(t)}. \tag{3}$$

On $(0, \infty)$ let $f_i(t) := F_i'(t)$ denote the (a.e.) density and define the hazard rate

$$\lambda_i(t) := \frac{f_i(t)}{1 - F_i(t)} \qquad (t > 0,\ F_i(t) < 1).$$

Using (3), we will repeatedly use the identity

$$z_i(t) = z_i(0+)\ \exp\Big(\int_0^t \lambda_i(u)\, du\Big), \qquad z_i(0+) := \frac{z_i(0)}{1 - F_i(0)}. \tag{4}$$

Since $F_i(\cdot)$ is continuous on $\mathbb{R}_+$, so is $z_i(t)$. Equipped with this, the players' indifference conditions are:

$$\lambda_C(t) = \frac{r(1-\alpha)}{2\alpha - 1} - \lambda_B(t) g_C^A(t), \tag{IC:A}$$

$$\lambda_C(t) = \frac{r(1-\alpha)}{2\alpha - 1} - \lambda_A(t) g_C^B(t), \tag{IC:B}$$

$$\lambda_A(t)(\pi_{AC} + g_B(t)) + \lambda_B(t)(1 + \pi_{AC} g_A(t)) = \frac{(1 + \pi_{AC}) r (1-\alpha)}{2\alpha - 1}, \tag{IC:C}$$

where we recall that $g_i(\cdot)$ (and resp. g_C^j) is probability of an immediate concession by player i (resp. C) in the induced bilateral continuation if the other peripheral concedes now.

Moreover, the complementary-slackness implications are:

$$\begin{aligned}
\lambda_C(t) + \lambda_B(t) g_C^A(t) > \lambda^{AG} &\implies \lambda_A(t) = 0,\\
\lambda_C(t) + \lambda_A(t) g_C^B(t) > \lambda^{AG} &\implies \lambda_B(t) = 0,\\
\lambda_A(t)\big(\pi_{AC} + g_B(t)\big) + \lambda_B(t)\big(1 + \pi_{AC} g_A(t)\big) > (1 + \pi_{AC})\lambda^{AG} &\implies \lambda_C(t) = 0.
\end{aligned}$$

For $0 \le s \le t$ define the survival function

$$\Lambda_i(t, s) := \exp\Big(-\int_s^t \lambda_i(u)\, du\Big), \qquad \Lambda_i(t) := \Lambda_i(t, 0).$$

Activation and terminal times. Let

$$t_i := \inf\{t > 0 : \; F_i(t) > F_i(0)\}$$

denote the first time at which player i concedes with positive density (i.e., after any atom at $t = 0$), and let

$$T_i := \inf\{t \geq 0 : \; z_i(t) = 1\}$$

denote the time at which i is believed behavioral with probability one on the no-concession path.

Proof roadmap. The proof of Proposition A.1 proceeds through lemmas that progressively pin down feasible posterior dynamics: (i) all players exhaust their concession mass at a common finite time T; (ii) on any interval strictly before T, at most one player can be inactive (have constant F_i); (iii) whenever F_i is strictly increasing on an interval, player i's local indifference holds a.e. on that interval; (iv) once a player starts conceding, they do not stop before T; (v) a collection of posterior-ordering lemmas rules out all strict orderings of (z_A, z_B, z_C) after all three players are active, forcing $z_A = z_B = z_C$ on that region; and (vi) the time-0 atom(s) are pinned down by requiring posterior alignment at the moment the last player becomes active.

A.2. Proposition A.1 and its proof

We now state and prove a complete characterization of equilibrium behavior in the no-concession subgame, summarized in Proposition A.1. For expositional clarity, we maintain the labeling convention from the main text and assume $z_A(0) \geq z_B(0)$.

A key structural feature of the equilibrium is that whenever the most reputable peripheral initially strictly dominates the other two players (i.e. $z_A(0) > \max\{z_B(0), z_C(0)\}$), that peripheral optimally remains *inactive* at the outset. Intuitively, as long as A is more reputable than B and C, A can wait without jeopardizing his bargaining position, while the other two players must adjust: because any concession by C resolves *both* negotiations, the local incentives of the remaining active peripheral are distorted and he must concede at a higher hazard rate than in a stand-alone AG interaction.

Formally, in this region equilibrium begins with an *initial two-player phase* on an interval $(0, t_A)$ during which A does not concede, while B and C concede with constant hazard rates

$$\lambda_B = (1 + \pi_{AC})\lambda^{AG} \quad \text{and} \quad \lambda_C = \lambda^{AG}, \qquad \text{where} \qquad \lambda^{AG} = \frac{r(1-\alpha)}{2\alpha - 1}.$$

The phase ends at the *activation time* t_A, defined as the first time at which the last inactive player (A) becomes willing to concede with positive density. At that moment all three players must be jointly active; by the posterior-ordering lemmas proved below, joint activity forces *posterior alignment*. Hence the identity and size of any time-0 atom(s) are pinned down by the requirement that the posteriors of the two active players reach $z_A(0)$ *simultaneously* at the activation time:

$$z_B(t_A) = z_C(t_A) = z_A(0).$$

Equivalently, the time-0 atom is chosen so that, under the initial-phase hazard rates, B and C "catch up" to A's prior reputation at the same calendar time.

To express this condition, define the "catch-up times absent atoms"

$$\tilde{t}_C := \frac{1}{\lambda^{AG}} \log\left(\frac{z_A(0)}{z_C(0)}\right), \qquad \tilde{t}_B := \frac{1}{(1+\pi_{AC})\lambda^{AG}} \log\left(\frac{z_A(0)}{z_B(0)}\right).$$

Comparing $\tilde{t}_C$ and $\tilde{t}_B$ determines which player (if any) must place an atom at $t = 0$ to ensure posterior alignment at activation: if $\tilde{t}_C \geq \tilde{t}_B$, C requires a (possibly zero) time-0 atom; if $\tilde{t}_B > \tilde{t}_C$, B requires a time-0 atom; and if $\tilde{t}_B = \tilde{t}_C$, no atom is required. After t_A, all three players concede with the common constant hazard λ^{AG} until the common terminal time at which posteriors reach 1

Proposition A.1 below presents a more complete version of Proposition 1 that describes the equilibrium dynamics along with initial atoms.

Proposition A.1 (Full version of Proposition 1)**.** *There is a unique equilibrium. Assume, without loss of generality, $z_A(0) \geq z_B(0)$. The equilibrium concession behavior on the no-concession path is:*

1. *There exists a finite time $T < \infty$ such that $T_i = T$ for all $i \in \{A, B, C\}$.*

2. *Once all three players concede with positive density, their hazards coincide and are constant:*
$$\lambda_A(t) = \lambda_B(t) = \lambda_C(t) = \lambda^{AG} \quad \textit{for a.e. such } t.$$

3. *If $z_C(0) \geq z_A(0)$ (the center is initially at least as reputable as both peripherals), then $F_C(0) = 0$ and the peripherals place time-0 mass so that posteriors align immediately:*
$$F_i(0) = \max\left\{1 - \frac{z_i(0)}{z_C(0)}, 0\right\} \quad (i \in \{A, B\}), \qquad \textit{hence} \qquad z_A(0+) = z_B(0+) = z_C(0).$$
All three players start conceding at time 0 with hazard rate λ^{AG}.

4. *If $z_A(0) = z_B(0) > z_C(0)$, then $F_C(0) = 1 - \frac{z_C(0)}{z_A(0)}$ and $F_A(0) = F_B(0) = 0$, so that $z_C(0+) = z_A(0) = z_B(0)$. All three players then concede from time 0 with hazard rate λ^{AG}.*

5. *If $z_A(0) = z_B(0) = z_C(0)$, then $F_A(0) = F_B(0) = F_C(0) = 0$ and all three players concede from time 0 with hazard rate λ^{AG}.*

6. *If $z_A(0) > z_B(0) \geq z_C(0)$, then $F_A(0) = F_B(0) = 0$ and C concedes at $t = 0$ with probability*
$$F_C(0) = 1 - \frac{z_C(0)}{z_A(0)^{\frac{\pi_{AC}}{1+\pi_{AC}}} z_B(0)^{\frac{1}{1+\pi_{AC}}}}.$$
Moreover, there is an initial phase $(0, t_A)$ in which A is inactive and (B, C) are active with hazard rates $(1+\pi_{AC})\lambda^{AG}$ and λ^{AG}, respectively, until their posteriors reach $z_A(0)$ at $t_A = \tilde{t}_B$; afterwards all three concede with hazard λ^{AG}.

7. *If $z_A(0) > z_C(0) > z_B(0)$, then A is initially inactive and (B, C) are active in an initial phase as in item 6. At most one of B and C has a time-0 atom, determined by comparing $\tilde{t}_C$ and $\tilde{t}_B$:*

   (a) *If $\tilde{t}_C > \tilde{t}_B$, then C has the atom and B does not:*

$$F_C(0) = 1 - \frac{z_C(0)}{z_A(0)^{\frac{\pi_{AC}}{1+\pi_{AC}}} z_B(0)^{\frac{1}{1+\pi_{AC}}}}, \qquad F_A(0) = F_B(0) = 0.$$

   (b) *If $\tilde{t}_B > \tilde{t}_C$, then B has the atom and C does not:*

$$F_B(0) = 1 - \frac{z_B(0)\, z_A(0)^{\pi_{AC}}}{z_C(0)^{1+\pi_{AC}}}, \qquad F_A(0) = F_C(0) = 0.$$

   In either subcase, posteriors align when A becomes active at $t_A = \min\{\tilde{t}_B, \tilde{t}_C\}$, and thereafter all three concede with hazard rate λ^{AG}.

Lemma 2. *If F is an equilibrium, then $T_i = T < \infty$ for all $i \in N$.*

Proof. The proof proceeds in three steps. We first show that if $T_i < \infty$ for some $i \in \{A, B, C\}$, then $T_j < \infty$, for all $j \neq i$. We then show that if $T_i < \infty$ for all i, then $T_i = T$ for all i. Finally, we show that $T_i < \infty$ for all i.

Suppose first $T_C < \infty$. Then, for any $t > T_C$, A and B strictly prefer to concede immediately. Hence,

$$T_C \geq \max\{T_A, T_B\}. \tag{5}$$

Suppose next $T_i < \infty$ for some $i \in \{A, B\}$. If i conceded by T_i, the continuation reduces to the bilateral AG interaction between j and C, and hence, $T_j = T_C < \infty$. By definition of T_i, i never concedes after T_i on the "no-concession path." Then at any time $t > T_i$, the continuation reduces to the bilateral AG interaction between j and C, but with the feature that a concession by C terminates both negotiations. Hence, local indifference conditions in that continuation therefore imply that for any $t \in (T_A, T]$, the players' instantaneous rates of concession are given by:

$$\lambda_j(t) = \begin{cases} (1 + \pi_{AC})\, \lambda^{AG} & \text{if } j = B, \\ \dfrac{1 + \pi_{AC}}{\pi_{AC}}\, \lambda^{AG} & \text{if } j = A, \end{cases} \quad \text{and } \lambda_C(t) = \lambda^{AG}.$$

Hence, $T_j = T_C < \infty$.

We next show that if $T_i < \infty$ for all i, terminal times must coincide. Suppose, toward a contradiction, that $T_i \neq T_j$ for some $i \neq j$. By (5), the only possible strict inequality is that one peripheral reaches 1 earlier than the other two. Thus, without loss, assume

$$T_A < T_B = T_C =: T.$$

Fix any $\Delta > 0$ sufficiently small such that $T_A + \Delta < T$. By definition of T_A as the *first* time $z_A(t) = 1$, player A concedes with positive probability on every interval $(t, T_A]$ with $t < T_A$. In particular, A must be willing to concede arbitrarily close to T_A.

We now show that, conditional on reaching T_A with no concession, A strictly prefers to wait an additional Δ rather than concede at T_A, contradicting the preceding paragraph.

Conditional on no concession up to T_A, player A's concession at T_A yields payoff $(1-\alpha)\pi_{AC}$. Consider instead the deviation: do not concede on $(T_A, T_A+\Delta]$ and (if still no concession has occurred by $T_A+\Delta$) concede at $T_A+\Delta$. On (T_A, T), player A is believed behavioral with probability one (since $z_A = 1$), hence A never concedes on-path after T_A; the continuation reduces to the two-player interaction between B and C, but with the key feature that a concession by C terminates *both* negotiations. The local indifference conditions in that continuation therefore imply that on (T_A, T) the hazard rates are constant and satisfy

$$\lambda_C(t) = \lambda^{AG} \quad \text{and} \quad \lambda_B(t) = (1+\pi_{AC})\lambda^{AG} \qquad \text{for a.e. } t \in (T_A, T).$$

Moreover, since $z_A(T_A) = 1$, the AG continuation following a concession by B at time $t \in (T_A, T)$ features an immediate concession by C to A with strictly positive probability $g_C^A(t) > 0$. By continuity of posteriors (and of the AG atom as a function of posteriors), there exists $\underline{g} > 0$ such that

$$g_C^A(t) \geq \underline{g} \quad \text{for all } t \in [T_A, T_A+\Delta].$$

Let $\mathcal{E}$ be the event that within $(T_A, T_A+\Delta]$ either (a) C concedes, or (b) B concedes and, in the induced continuation, C concedes to A immediately with probability at least $\underline{g}$. Under the hazard rate representation, conditional on no concession up to T_A, the probability that C concedes in $(T_A, T_A+\Delta]$ is

$$\int_{T_A}^{T_A+\Delta} \lambda_B(s, T_A)\lambda_C(s, T_A)\, \lambda_C(s)\, ds,$$

and the probability that B concedes in $(T_A, T_A+\Delta]$ is

$$\int_{T_A}^{T_A+\Delta} \lambda_B(s, T_A)\lambda_C(s, T_A)\, \lambda_B(s)\, ds.$$

Using $\lambda_i(s, T_A) \geq 1 - \int_{T_A}^{s} \lambda_i(u)\, du$ and constancy of λ_B, λ_C on this interval, we obtain the lower bound

$$\Pr(\mathcal{E}) \;\geq\; \Delta\big(\lambda_C + \underline{g}\,\lambda_B\big) - o(\Delta),$$

where $o(\Delta)/\Delta \to 0$ as $\Delta \downarrow 0$.

Under the deviation, conditional on reaching T_A,

- on $\mathcal{E}$, player A is conceded to and obtains payoff at least $\alpha\pi_{AC}$;
- on $\mathcal{E}^c$, A concedes at $T_A+\Delta$ and obtains $(1-\alpha)\pi_{AC}$.

Discounting only *reduces* payoffs under the deviation relative to evaluating them at T_A, so a lower bound on the deviation payoff is obtained by discounting *all* payoffs by $e^{-r\Delta}$. Hence, conditional on reaching T_A, the deviation payoff is at least

$$e^{-r\Delta}\pi_{AC}\Big((1-\alpha)\Pr(\mathcal{E}^c) + \alpha\Pr(\mathcal{E})\Big) = e^{-r\Delta}\pi_{AC}\Big((1-\alpha) + (2\alpha-1)\Pr(\mathcal{E})\Big).$$

Subtracting the immediate-concession payoff $(1-\alpha)\pi_{AC}$ and using $e^{-r\Delta} \geq 1 - r\Delta$ yields

$$\text{gain from waiting } \geq \pi_{AC}\Big((2\alpha-1)\Pr(\mathcal{E}) - r\Delta(1-\alpha)\Big).$$

Using the lower bound $\Pr(\mathcal{E}) \geq \Delta(\lambda_C + \underline{g}\,\lambda_B) - o(\Delta)$ and $\lambda_C = \lambda^{AG}$ gives

$$(2\alpha-1)\Pr(\mathcal{E}) - r\Delta(1-\alpha) \;\geq\; \Delta(2\alpha-1)\underline{g}\,\lambda_B - o(\Delta) \;=\; \Delta\underline{g}(2\alpha-1)(1+\pi_{AC})\lambda^{AG} - o(\Delta).$$

For $\Delta > 0$ sufficiently small, the right-hand side is strictly positive. Hence A strictly prefers to wait, contradicting that A concedes with positive probability arbitrarily close to T_A. Therefore, if at least one terminal time is finite, the terminal times must coincide.

Suppose finally that $T_A = T_B = T_C = \infty$. Fix $\varepsilon \in (0, z_C(0))$ and define

$$t_2 := \inf\{t \geq 0 : \; F_C(t) \geq 1 - z_C(0) - 2\varepsilon\}, \qquad t_1 := \inf\{t \geq 0 : \; F_C(t) \geq 1 - z_C(0) - \varepsilon\}.$$

Continuity of F_C implies $t_2 < t_1 < \infty$. By construction,

$$F_C(t_1) - F_C(t_2) \leq \varepsilon.$$

Consider player A's continuation problem at time t_2 conditional on no concession up to t_2. If A concedes immediately at t_2, he obtains $(1-\alpha)\pi_{AC}$ (evaluated at t_2). If instead A commits to wait until t_1 and then (if necessary) concede, then the most favorable outcomes for A over $(t_2, t_1]$ arise from being conceded to by C; this can happen with (conditional) probability at most ε by the preceding inequality. Therefore A's continuation payoff from waiting until t_1 is bounded above by

$$\varepsilon\alpha\pi_{AC} + e^{-r(t_1-t_2)}\Big(\varepsilon\alpha\pi_{AC} + (1-\varepsilon)(1-\alpha)\pi_{AC}\Big).$$

For $\varepsilon > 0$ sufficiently small, this upper bound is strictly less than $(1-\alpha)\pi_{AC}$. Thus A strictly prefers conceding at t_2 to waiting until t_1, contradicting equilibrium. Hence $T < \infty$. □

Lemma 3. *Let $0 < t' < t < T$. If $F_i(t) = F_i(t')$ for some $i \in N$, then for each $j \in N \setminus \{i\}$,*

$$F_j(t) > F_j(t').$$

Equivalently, on any interval strictly before T, at most one player can be inactive.

Proof. Fix $0 < t' < t < T$ and suppose $F_i(t) = F_i(t')$.

Suppose there exists $j \neq i$ with $F_j(t) = F_j(t')$ and suppose the remaining player $k \in N \setminus \{i, j\}$ assigns positive concession probability on $(t', t]$. Consider k's strategy restricted to histories in which no concession occurs before t'. Holding fixed all play outside $(t', t]$, define a deviation for player k that shifts *all* concession probability that k assigns to times in $(t', t]$ to an atom at time t'. Because players i and j are inactive on $(t', t]$, this deviation does not alter the probability distribution over *who concedes first* conditional on reaching t'; it only (weakly) reduces the calendar time of agreement. Since payoffs are discounted in calendar time and the division conditional on who concedes is unchanged, the deviation strictly increases k's expected payoff whenever k concedes with positive probability on $(t', t]$. This contradicts optimality.

If all three F_A, F_B, F_C were constant on $[t', t]$, then since $t < T$ there exists some $t'' > t$ at which some player concedes with positive probability. The same time-shifting argument (moving that concession probability to an atom at t') yields a strict improvement, again contradicting equilibrium.

Therefore, if F_i is constant on $[t', t]$, both remaining players must strictly increase on that interval.

□

Lemma 4. *Suppose that $F_i(\cdot)$ is strictly increasing on $[t_1, t_2]$. Then, the indifference condition for player i holds at a.e. $t \in (t_1, t_2)$.*

Proof. **Case 1: $i = A$.**

Define

$$S_A(s; t_1) := \lambda_B(s, t_1)\, \lambda_C(s, t_1) = \exp\Big(- \int_{t_1}^{s} \big(\lambda_B(u) + \lambda_C(u)\big)\, du \Big),$$
$$R_A(s) := \lambda_C(s)\, \alpha + \lambda_B(s) \Big(g_C^A(s) \alpha + (1 - g_C^A(s))(1 - \alpha) \Big),$$

where we can interpret $S_A(s; t_1)$ as the joint survival from t_1 to s (i.e., no concession by B or C on $(t_1, s]$ and $R_A(s)$ as the instantaneous expected "reward rate" (in units of π_{AC}) from the first event at time s. Fix t_1 and define $u_A(t)$ as A's continuation payoff at time t_1 from conceding at exactly time $t \in [t_1, t_2]$, conditional on no concession strictly before t:

$$\frac{u_A(t)}{\pi_{AC}} = \int_{t_1}^{t} \exp\big(- r(s - t_1) \big)\, S_A(s; t_1)\, R_A(s)\, ds \; + \; \exp\big(- r(t - t_1) \big)\, S_A(t; t_1)\, (1 - \alpha), \qquad t \in [t_1, t_2].$$

By piecewise continuity of $\lambda_B(s), \lambda_C(s)$ and boundedness of the integrands, $u_A(\cdot)$ is continuous on $[t_1, t_2]$.

Let $M := \arg\max_{w \in [t_1, t_2]} u_A(w)$ be the set of maximizers. If A assigns positive concession density at some $t \in (t_1, t_2)$, then $t \in M$ by optimality. Since F_A is strictly increasing on $[t_1, t_2]$, its support is dense in (t_1, t_2), hence M is dense in (t_1, t_2). A continuous function that attains its maximum on a dense set must be constant; therefore $u_A(t)$ is constant on $[t_1, t_2]$.

Since u_A is absolutely continuous (indeed differentiable a.e.) with derivative obtained by the fundamental theorem of calculus, differentiating $u_A(t)$ a.e. on (t_1, t_2) and setting $u_A'(t) = 0$ yields (IC:A) a.e. on (t_1, t_2).

(t_1, t_2).

Case 2: $i = C$. Fix t_1. For each $t \in [t_1, t_2]$, let $u_C(t)$ denote C's continuation payoff *evaluated at time t_1 and conditional on no concession up to t_1*, when C follows the strategy: "concede at time t if no player concedes before t."

Define similarly

$$
\begin{aligned}
S_C(s;t_1) &:= \lambda_A(s,t_1)\,\lambda_B(s,t_1) = \exp\Bigl(-\int_{t_1}^{s} \bigl(\lambda_A(u)+\lambda_B(u)\bigr)\,du\Bigr),\\
V_C^A(s) &:= \alpha\,\pi_{AC} \;+\; \Bigl(g_B(s)\alpha + (1-g_B(s))(1-\alpha)\Bigr),\\
V_C^B(s) &:= \alpha \;+\; \pi_{AC}\Bigl(g_A(s)\alpha + (1-g_A(s))(1-\alpha)\Bigr),\\
R_C(s) &:= \lambda_A(s)\,V_C^A(s) + \lambda_B(s)\,V_C^B(s).
\end{aligned}
$$

We can then write C's continuation payoff from following this strategy (to concede at t if nobody else has by then) as:

$$
u_C(t) = \int_{t_1}^{t} e^{-r(s-t_1)}\,S_C(s;t_1)\,R_C(s)\,ds \;+\; e^{-r(t-t_1)}\,S_C(t;t_1)\,(1+\pi_{AC})(1-\alpha), \qquad t \in [t_1,t_2].
$$

The same density-of-support argument implies $u_C(\cdot)$ is constant on $[t_1,t_2]$, hence $u_C'(t)=0$ a.e. and therefore (IC:C) holds a.e. on (t_1,t_2).

□

Lemma 5. *If $F_C(t)>0$, then $F_C(\cdot)$ is strictly increasing on $[t,T]$. Therefore,* (IC:C) *holds for a.e. $s \in [t,T]$.*

Proof. Assume $F_C(t)>0$ for some $t<T$ and suppose, toward a contradiction, that F_C is not strictly increasing on $[t,T]$. Then there exist $t<t'<T$ such that $F_C(t)=F_C(t')>0$.

Let

$$
\tau := \inf\{s \le t:\; F_C(s)=F_C(t')\}
$$

be the left endpoint of the first plateau at level $F_C(t')$. Suppose for now $\tau>0$. By construction, F_C is constant on $[\tau,t']$ and strictly increasing on $(\tau-\varepsilon,\tau)$ for every sufficiently small $\varepsilon>0$ (by continuity).

Step 1: on (τ,t'), both peripherals are active and satisfy their ICs. Because F_C is constant on $[\tau,t']$, Lemma 3 implies that both F_A and F_B are strictly increasing on $[\tau,t']$. Hence, by Lemma 4, (IC:A) and (IC:B) hold for a.e. $s\in(\tau,t')$. Since $\lambda_C(s)=0$ a.e. on (τ,t'), these equalities imply

$$
\lambda_B(s)g_C^A(s)=\lambda^{AG} \quad \text{and} \quad \lambda_A(s)g_C^B(s)=\lambda^{AG} \qquad \text{for a.e. } s\in(\tau,t').
$$

In particular, $g_C^A(s), g_C^B(s)>0$, and therefore, $\lambda_A(s),\lambda_B(s)>\lambda^{AG}$ a.e. on (τ,t').

Step 2: C strictly prefers not to concede at τ, contradicting that C concedes before τ.

Let $V_C(\tau)$ denote C's equilibrium continuation value at time τ conditional on no concession up to τ. Note that if C concedes immediately at τ, her payoff equals

$$
u_C(\tau,\tau)=(1+\pi_{AC})(1-\alpha).
$$

We show that $V_C(\tau)>(1+\pi_{AC})(1-\alpha)$, implying that C strictly prefers to wait at τ.

Consider waiting for a small $\varepsilon>0$. Over $(\tau,\tau+\varepsilon]$, either A concedes, or B concedes, or neither concedes. Using only the *minimal* payoff C can guarantee in each event gives a lower bound:

- if A concedes in $(\tau, \tau+\varepsilon]$, then C obtains at least $\alpha\pi_{AC}$ in the negotiation between A and C and at least $(1-\alpha)$ in the negotiation between B and C;
- if B concedes in $(\tau, \tau+\varepsilon]$, then C obtains at least α in the negotiation between B and C and at least $(1-\alpha)\pi_{AC}$ in the negotiation between A and C;
- if no concession occurs in $(\tau, \tau+\varepsilon]$, then at $\tau+\varepsilon$ player C can still secure $(1+\pi_{AC})(1-\alpha)$ by conceding immediately.

Discounting over ε satisfies $e^{-r\varepsilon} \geq 1 - r\varepsilon$. Therefore,

$$V_C(\tau) \geq \Big(\int_\tau^{\tau+\varepsilon} \lambda_A(u)\,du\Big)(\alpha\pi_{AC} + 1 - \alpha) + \Big(\int_\tau^{\tau+\varepsilon} \lambda_B(u)\,du\Big)(\alpha + \pi_{AC}(1-\alpha)) \\ + (1 - r\varepsilon)\Big(1 - \int_\tau^{\tau+\varepsilon} \lambda_A(u)\,du - \int_\tau^{\tau+\varepsilon} \lambda_B(u)\,du\Big)(1+\pi_{AC})(1-\alpha).$$

Since $\lambda_A((\cdot)), \lambda_B((\cdot)) > \lambda^{AG}$ for a.e. $s \in (\tau, \tau+\varepsilon]$, expanding the right-hand side and writing $P := (1+\pi_{AC})(1-\alpha)$ gives

$$V_C(\tau) \geq P + (2\alpha - 1)\Big(\pi_{AC}\int_\tau^{\tau+\varepsilon} \lambda_A(u)\,du + \int_\tau^{\tau+\varepsilon} \lambda_B(u)\,du\Big) - r\varepsilon\, P + O(\varepsilon^2) \\ > P + (2\alpha-1)(1+\pi_{AC})\lambda^{AG}\varepsilon - r\varepsilon\, P + O(\varepsilon^2) = P + O(\varepsilon^2),$$

where the last equality uses $(2\alpha - 1)\lambda^{AG} = r(1-\alpha)$. For $\varepsilon > 0$ sufficiently small, $V_C(\tau) > P = (1+\pi_{AC})(1-\alpha)$.

But $(1+\pi_{AC})(1-\alpha)$ is precisely the payoff C obtains by conceding at τ. Therefore, C strictly prefers to wait rather than concede at τ. By continuity of payoffs in time, C also strictly prefers to wait rather than concede at any time in a left-neighborhood of τ, contradicting that F_C is strictly increasing just before τ.

This contradiction shows that no plateau can occur after F_C becomes positive. Hence F_C is strictly increasing on $[t, T]$.

Suppose $\tau = 0$. Then $F_C(0) = F_C(t') > 0$ and F_C is constant on $(0, t']$, so $\lambda_C(t) = 0$ for a.e. $t \in (0, t']$. Fix $\varepsilon \in (0, t']$ and set $t_0 = \varepsilon/2$. Since F_C is constant on $[t_0, \varepsilon]$, Lemma 3 implies $F_A(\varepsilon) > F_A(t_0)$ and $F_B(\varepsilon) > F_B(t_0)$, hence $\lambda_A, \lambda_B > 0$ a.e. on $(0, \varepsilon)$. By Lemma 4, (IC:A) and (IC:B) hold a.e. on (t_0, ε); with $\lambda_C = 0$ this implies $\lambda_B g_C^A = \lambda_A g_C^B = \lambda^{AG}$ and therefore $\lambda_A, \lambda_B > \lambda^{AG}$ a.e. on (t_0, ε).

Consider C's deviation at time 0: wait until time ε and (if still unresolved) concede at ε. Let $S_C(s;0) := \lambda_A(s,0)\lambda_B(s,0)$. The deviation payoff is bounded below by

$$\int_0^\varepsilon e^{-rs} S_C(s;0)\Big[\lambda_A(s)(\alpha\pi_{AC}+1-\alpha) + \lambda_B(s)(\alpha+\pi_{AC}(1-\alpha))\Big]ds + e^{-r\varepsilon} S_C(\varepsilon;0)(1+\pi_{AC})(1-\alpha),$$

which exceeds $(1+\pi_{AC})(1-\alpha)$ for $\varepsilon > 0$ sufficiently small since $\lambda_A, \lambda_B > \lambda^{AG}$. Thus C strictly prefers waiting to conceding at time 0, contradicting $F_C(0) > 0$.

The final claim follows from Lemma 4.

□

Lemma 6. *Fix $i \in \{A, B\}$. If for some $t < T$,*

$$z_i(t) > z_i(0) \quad \text{and} \quad z_i(t) > z_C(t),$$

then for every $t' > t$ such that $z_i(t') \geq z_C(t')$, we have $F_i(t') > F_i(t)$. Equivalently, F_i cannot be constant on any interval $[t, t'] \subset (0, T)$ with $z_i(t') \geq z_C(t')$.

Proof. We prove the claim for $i = A$ (the case $i = B$ is symmetric). Assume $z_A(t) > z_A(0)$ and $z_A(t) > z_C(t)$.

Suppose, toward a contradiction, that there exists $t' > t$ such that

$$F_A(t') = F_A(t) \qquad \text{and} \qquad z_A(t') \geq z_C(t').$$

Since F_A is constant on $[t, t']$, we have $z_A(s) = z_A(t)$ for all $s \in [t, t']$. Since $z_C(\cdot)$ is weakly increasing, $z_C(s) \leq z_C(t')$ for all $s \in [t, t']$. Therefore,

$$z_A(s) = z_A(t') \geq z_C(t') \geq z_C(s) \qquad \text{for all } s \in [t, t']. \tag{†}$$

Let

$$\tau := \inf\{s \leq t : \; F_A(s) = F_A(t)\}.$$

Then F_A is constant on $[\tau, t']$. Moreover, since $z_A(t) > z_A(0)$, we have $F_A(t) > 0$ and (by continuity at positive times) $\tau \in (0, t]$. Hence F_A is not constant on any left-neighborhood of τ.

Because F_A is constant on $[\tau, t']$, Lemma 3 implies that both F_B and F_C are strictly increasing on $[\tau, t']$. Hence $\lambda_B, \lambda_C > 0$ a.e. on (τ, t'), and by Lemma 4, (IC:B) and (IC:C) hold a.e. on (τ, t'). Since F_A is constant on (τ, t'), we have $\lambda_A = 0$ a.e. on (τ, t'), so (IC:B) yields

$$\lambda_C(u) = \lambda^{AG} \qquad \text{for a.e. } u \in (\tau, t'). \tag{‡}$$

Next, by (†) applied on $[\tau, t]$, we have $z_A(u) > z_C(u)$ for all $u \in [\tau, t]$ (since $z_A(t) > z_C(t)$), so in the AG continuation following B's concession at time u we have

$$g_C^A(u) > 0 \quad \text{and} \quad g_A(u) = 0 \qquad \text{for all } u \in [\tau, t].$$

Using $\lambda_A = 0$ and $g_A = 0$ in (IC:C) therefore implies

$$\lambda_B(u) = (1 + \pi_{AC})\lambda^{AG} \qquad \text{for a.e. } u \in (\tau, t). \tag{$\star$}$$

By continuity of posteriors on $(0, T)$ and the fact that $g_C^A(u) = \max\{1 - z_C(u)/z_A(u), 0\}$ in the AG continuation, there exist $\varepsilon > 0$ and $\underline{g} > 0$ such that $\tau + \varepsilon \leq t$ and

$$g_C^A(u) \geq \underline{g} \qquad \text{for all } u \in [\tau, \tau + \varepsilon]. \tag{$\star\star$}$$

Now, let us first suppose that $\tau > 0$. Since F_A is not constant on any left-neighborhood of τ, there exists a sequence $s_n \uparrow \tau$ such that $\lambda_A(s_n) > 0$ (at points of continuity). Consider such an $s = s_n$ close enough to τ. Using (‡), ($\star$), and ($\star\star$), the same ε-deviation argument as in Lemma 2 implies that, conditional on reaching time s with no concession, player A

strictly prefers to wait until $s+\varepsilon$ (and concede then if necessary) rather than concede at time s: over $(s, s+\varepsilon]$, the effective rate at which A is conceded to is at least $\lambda_C + \lambda_B \underline{g} > \lambda^{AG}$, which yields a strictly positive first-order gain that dominates the $O(\varepsilon)$ discounting loss. This contradicts $\lambda_A(s) > 0$.

Now suppose that $\tau = 0$. Therefore, $z_A(0+) > z_A(0)$. Thus, $F_A(0) > 0$, i.e., there is a time 0 atom from A. Following the previous argument, the effective rate at which A is conceded to on $(0, \epsilon)$ is at least $\lambda_C + \lambda_B \underline{g} > \lambda^{AG}$, which yields a strict incentive for A to wait at 0. Therefore, there cannot be an atom at 0 by A contradicting $z_A(t) > z_A(0)$.

Therefore no such $t' > t$ can exist, and $F_A(t') > F_A(t)$ for all $t' > t$ with $z_A(t') \geq z_C(t')$.

□

Lemma 7. *Let $i, j \in \{A, B\}$ with $i \neq j$. If for some $u > 0$,*

$$z_i(u) > z_C(u) \geq z_j(u),$$

then $z_i(u) = z_i(0)$.

Proof. Without loss of generality, take $(i,j) = (A,B)$. Suppose $z_A(u) > z_C(u) \geq z_B(u)$ for some $u > 0$. Assume toward a contradiction that $z_A(u) > z_A(0)$.

Define

$$\tau := \sup\Big\{s \in [u, T) : \ 1 > z_A(t) > z_C(t) \geq z_B(t) \text{ for all } t \in [u, s]\Big\}.$$

By Lemma 6, F_A is strictly increasing on $[u, \tau]$, hence (IC:A) holds a.e. on (u, τ) by Lemma 4.

We consider two cases for λ_B on (u, τ).

Case 1: $\lambda_B > 0$ on a set of positive measure in (u, τ). Since λ_B is piecewise continuous, there exists a point $s \in (u, \tau)$ at which λ_B is continuous and $\lambda_B(s) > 0$, hence $\lambda_B > 0$ on some open interval $(s_1, s_2) \subset (u, \tau)$. On (s_1, s_2), F_B is strictly increasing, so (IC:B) holds a.e. there.

On (u, τ) we have $z_B \leq z_C$, hence the AG atom $g_C^B(\cdot) = 0$ throughout, while $z_A > z_C$ implies $g_C^A(\cdot) > 0$ throughout. Therefore, on (s_1, s_2), (IC:B) implies $\lambda_C = \lambda^{AG}$ a.e., whereas (IC:A) implies $\lambda_C = \lambda^{AG} - \lambda_B g_C^A < \lambda^{AG}$ a.e., a contradiction.

Case 2: $\lambda_B = 0$ a.e. on (u, τ). Then F_B is constant on (u, τ), so Lemma 3 implies that both F_A and F_C are strictly increasing on (u, τ). Hence (IC:A) and (IC:C) hold a.e. on (u, τ). Since $z_A > z_C$ on (u, τ), we have $g_A(\cdot) = 0$ there, and thus (IC:C) together with $\lambda_B = 0$ implies $\lambda_A = \frac{(1+\pi_{AC})}{\pi_{AC}} \lambda^{AG}$ a.e. on (u, τ), while (IC:A) implies $\lambda_C = \lambda^{AG}$ a.e. on (u, τ).

Therefore z_A grows strictly faster than z_C on (u, τ), so the strict inequality $z_A > z_C$ persists at τ and (by continuity) beyond τ unless $z_A(\tau) = 1$. If $z_A(\tau) = 1$, then $T_A < T$ contradicting Lemma 2. If $z_A(\tau) < 1$, then the strict inequality persists on a right-neighborhood of τ, contradicting the definition of τ as a supremum. Either way yields a contradiction.

Thus $z_A(u) > z_A(0)$ is impossible, so $z_A(u) = z_A(0)$. □

Lemma 8. *For all $t > 0$,*

$$\min\{z_A(t), z_B(t)\} \leq z_C(t).$$

Proof. Suppose, toward a contradiction, that $\min\{z_A(t), z_B(t)\} > z_C(t)$ for some $t > 0$. Define

$$\tau := \sup\Big\{s \geq t :\ 1 > z_A(u),\ z_B(u) > z_C(u) \text{ for all } u \in [t, s]\Big\}.$$

On $[t, \tau]$, both peripherals are strictly more reputable than C, hence in the AG continuation after C concedes we have $g_A(u) = g_B(u) = 0$ for all $u \in [t, \tau]$.

We claim F_C cannot be constant on (t, τ). If F_C were constant on (t, τ), then z_C would be constant there, while z_A and z_B are weakly increasing. In particular, the strict inequality $z_A, z_B > z_C$ would persist at τ. If $\max\{z_A(\tau), z_B(\tau)\} = 1$, then Lemma 2 is violated. If instead $1 > z_A(\tau), z_B(\tau) > z_C(\tau)$, continuity implies the same strict inequality holds on a right-neighborhood of τ, contradicting the definition of τ. Hence F_C is not constant on (t, τ).

Therefore there exists $s \in (t, \tau)$ with $F_C(s) > F_C(t)$. By Lemma 5, F_C is strictly increasing on $[s, T]$, so $\lambda_C > 0$ a.e. on (s, τ) and (IC:C) holds a.e. on (s, τ) by Lemma 4. Since $g_A = g_B = 0$ on (s, τ), (IC:C) simplifies to

$$\pi_{AC}\lambda_A(u) + \lambda_B(u) = (1 + \pi_{AC})\lambda^{AG} \qquad \text{for a.e. } u \in (s, \tau).$$

By Lemma 3, at least one of F_A, F_B is strictly increasing on (s, τ). We now show each possible activity pattern leads to a contradiction.

(i) Exactly one peripheral is active on (s, τ). Without loss, suppose F_A is strictly increasing and F_B is constant on (s, τ). Then $\lambda_B = 0$ a.e. on (s, τ), so the simplified condition yields $\lambda_A = (1 + \pi_{AC})\lambda^{AG}$ a.e. Moreover, since F_A is strictly increasing, (IC:A) holds a.e. on (s, τ), implying $\lambda_C = \lambda^{AG} - \lambda_B g_C^A = \lambda^{AG}$ a.e. on (s, τ). Hence z_A grows strictly faster than z_C on (s, τ), implying $z_A(\tau) > z_C(\tau) = z_B(\tau)$ and $z_A(\tau) > z_A(0)$. This contradicts Lemma 7.

(ii) Both peripherals are active on (s, τ). Then (IC:A) and (IC:B) hold a.e. on (s, τ), and because $z_A, z_B > z_C$ on (s, τ) we have $g_C^A, g_C^B > 0$ there. Hence (IC:A) and (IC:B) imply $\lambda_C < \lambda^{AG}$ a.e. on (s, τ), while the simplified condition fixes $\pi_{AC}\lambda_A + \lambda_B = (1 + \pi_{AC})\lambda^{AG}$. Therefore at least one of λ_A, λ_B exceeds λ^{AG} on a set of positive measure, implying at least one of z_A, z_B grows strictly faster than z_C on (s, τ). At τ, maximality of τ forces $\min\{z_A(\tau), z_B(\tau)\} = z_C(\tau)$ while the other peripheral strictly exceeds $z_C(\tau)$, and both peripherals have strictly increased from their priors on (s, τ). This contradicts Lemma 7 (applied to the peripheral that strictly exceeds z_C at τ).

Both cases are impossible, so $\min\{z_A(t), z_B(t)\} > z_C(t)$ cannot occur.

□

Lemma 9. *For all $t > 0$,*

$$z_C(t) \leq \max\{z_A(t), z_B(t)\}.$$

Proof. Suppose, toward a contradiction, that $z_C(t) > \max\{z_A(t), z_B(t)\}$ for some $t > 0$.

Case 1: $z_C(t) = z_C(0)$. Then F_C is constant on $(0, t)$ and hence $\lambda_C(s) = 0$ for a.e. $s \in (0, t)$. Since posteriors are weakly increasing in s, we have

$$z_C(s) > \max\{z_A(s), z_B(s)\} \qquad \text{for all } s \in (0, t].$$

By Lemma 3, both F_A and F_B are strictly increasing on $(0,t)$. Therefore, by Lemma 4, (IC:A) and (IC:B) hold for a.e. $s \in (0,t)$.

Fix any such $s \in (0,t)$. Since $z_C(s) > z_A(s)$ and $z_C(s) > z_B(s)$, player C is the strong player in the induced AG continuation following either peripheral's concession at time s. Hence

$$g_C^A(s) = g_C^B(s) = 0.$$

Substituting into (IC:A) (or (IC:B)) yields $\lambda_C(s) = \lambda^{AG}$ for a.e. $s \in (0,t)$, contradicting $\lambda_C(s) = 0$ there.

Case 2: $z_C(t) > z_C(0)$. Define

$$\tau := \sup\Big\{s \geq t : \ 1 > z_C(u) > \max\{z_A(u), z_B(u)\} \text{ for all } u \in [t,s]\Big\}.$$

Then $\tau > t$ and, by continuity of posteriors on $(0,T)$,

$$z_C(u) > \max\{z_A(u), z_B(u)\} \qquad \text{for all } u \in (t,\tau).$$

Since $z_C(t) > z_C(0)$, we have $F_C(t) > 0$, and Lemma 5 implies that F_C is strictly increasing on (t,τ). Hence, by Lemma 4, (IC:C) holds for a.e. $u \in (t,\tau)$.

Step 1: at least one peripheral accumulates strictly less than $\lambda^{AG}(\tau - t)$. Fix $u \in (t,\tau)$ such that (IC:C) holds. Since $z_C(u) > z_A(u)$ and $z_C(u) > z_B(u)$, in the AG continuation induced by C's concession at time u, both peripherals are weak against C, so

$$g_A(u) > 0 \quad \text{and} \quad g_B(u) > 0.$$

Thus (IC:C) reads

$$\lambda_A(u)\big(\pi_{AC} + g_B(u)\big) + \lambda_B(u)\big(1 + \pi_{AC} g_A(u)\big) = (1 + \pi_{AC})\lambda^{AG},$$

with $\pi_{AC} + g_B(u) > \pi_{AC}$ and $1 + \pi_{AC} g_A(u) > 1$. Therefore, whenever $(\lambda_A(u), \lambda_B(u)) \neq (0,0)$, we have the strict inequality

$$\pi_{AC}\,\lambda_A(u) + \lambda_B(u) < (1 + \pi_{AC})\lambda^{AG}.$$

By Lemma 3, F_A and F_B cannot both be constant on any subinterval of (t,τ), and since hazard rates are piecewise continuous, this implies $(\lambda_A(u), \lambda_B(u)) \neq (0,0)$ for a.e. $u \in (t,\tau)$. Integrating over (t,τ) yields

$$\pi_{AC} \int_t^\tau \lambda_A(u)\,du + \int_t^\tau \lambda_B(u)\,du < (1 + \pi_{AC})\lambda^{AG}(\tau - t),$$

and hence

$$\min\Big\{\int_t^\tau \lambda_A(u)\,du,\ \int_t^\tau \lambda_B(u)\,du\Big\} < \lambda^{AG}(\tau - t). \tag{†}$$

Step 2: $\lambda_C = \lambda^{AG}$ a.e. on (t,τ) and $z_C(\tau) > \min\{z_A(\tau), z_B(\tau)\}$. On (t,τ) we have $z_C > \max\{z_A, z_B\}$, so as in Case 1,

$$g_C^A(u) = g_C^B(u) = 0 \qquad \text{for all } u \in (t,\tau).$$

Therefore, whenever $\lambda_A(u) > 0$, (IC:A) implies $\lambda_C(u) = \lambda^{AG}$; whenever $\lambda_B(u) > 0$, (IC:B) implies $\lambda_C(u) = \lambda^{AG}$. Again using Lemma 3 and piecewise continuity, at least one of λ_A, λ_B is positive a.e. on (t, τ), hence

$$\lambda_C(u) = \lambda^{AG} \qquad \text{for a.e. } u \in (t, \tau).$$

Thus $\int_t^\tau \lambda_C(u)\, du = \lambda^{AG}(\tau - t)$. Combining with (†) and the fact $z_C(t) > z_A(t), z_B(t)$ implies

$$z_C(\tau) > \min\{z_A(\tau), z_B(\tau)\}. \tag{‡}$$

Step 3: contradiction at τ and beyond. By definition of τ and continuity, either (i) $z_C(\tau) = 1$, or (ii) $z_C(\tau) = \max\{z_A(\tau), z_B(\tau)\}$. Case (i) is impossible because all posteriors reach 1 only at the common terminal time T. Hence we are in case (ii). Together with (‡), this implies (without loss of generality)

$$z_C(\tau) = z_A(\tau) > z_B(\tau). \tag{$\star$}$$

Since $z_C(u) > z_A(u)$ for all $u \in [t, \tau)$ while $z_C(\tau) = z_A(\tau)$, it follows that $z_A(\tau) > z_A(0)$.

We claim that

$$z_C(u) \geq \max\{z_A(u), z_B(u)\} \qquad \text{for all } u \in [\tau, T). \tag{$\star\star$}$$

If $z_A(u) > z_C(u)$ for some $u \geq \tau$, then Lemma 8 implies $z_B(u) \leq z_C(u)$, so $z_A(u) > z_C(u) \geq z_B(u)$, and Lemma 7 forces $z_A(u) = z_A(0)$, contradicting $z_A(u) \geq z_A(\tau) > z_A(0)$. If instead $z_B(u) > z_C(u)$ for some $u \geq \tau$, then similarly $z_B(u) > z_C(u) \geq z_A(u)$ and Lemma 7 forces $z_B(u) = z_B(0)$, but z_B is nondecreasing and $z_B(\tau) < z_C(\tau) \leq z_C(u)$, a contradiction. This proves $(\star\star)$.

On (τ, T) we therefore have $g_C^A = g_C^B = 0$ and, by the same argument as in Step 2, $\lambda_C(u) = \lambda^{AG}$ for a.e. $u \in (\tau, T)$. Hence $\int_\tau^T \lambda_C(u)\, du = \lambda^{AG}(T - \tau)$.

Since $z_A(\tau) = z_C(\tau)$ and $z_A(T) = z_C(T) = 1$, we must have

$$\int_\tau^T \lambda_A(u)\, du = \int_\tau^T \lambda_C(u)\, du = \lambda^{AG}(T - \tau).$$

Since $z_B(\tau) < z_C(\tau)$ but $z_B(T) = z_C(T) = 1$, we must have

$$\int_\tau^T \lambda_B(u)\, du > \int_\tau^T \lambda_C(u)\, du = \lambda^{AG}(T - \tau).$$

Finally, (IC:C) holds for a.e. $u \in (\tau, T)$ by Lemma 5 and Lemma 4. Integrating (IC:C) over (τ, T) yields

$$\int_\tau^T \Big(\lambda_A(u)(\pi_{AC} + g_B(u)) + \lambda_B(u)(1 + \pi_{AC} g_A(u))\Big)\, du = (1 + \pi_{AC})\lambda^{AG}(T - \tau).$$

But $g_A, g_B \geq 0$, so the left-hand side is at least

$$\pi_{AC} \int_\tau^T \lambda_A(u)\, du + \int_\tau^T \lambda_B(u)\, du > (1 + \pi_{AC})\lambda^{AG}(T - \tau),$$

a contradiction. This completes the proof. □

Lemma 10. *If F is an equilibrium, then $\nexists\, t > 0$ such that $z_A(t) = z_C(t) > z_B(t)$.*

Proof. Suppose, toward a contradiction, that $z_A(t) = z_C(t) > z_B(t)$ for some $t > 0$. Fix $t' > t$ sufficiently close to t.

Step 1: none of F_A, F_B, F_C can be constant on (t, t').

- If F_C were constant on (t, t'), then by Lemma 3 both F_A and F_B would be strictly increasing there, hence for some $s \in (t, t')$ we would have $z_A(s) > z_C(s) > z_B(s)$, contradicting Lemma 7.

- If F_A were constant on (t, t'), then F_B and F_C would be strictly increasing there, implying $z_C(s) > \max\{z_A(s), z_B(s)\}$ for some $s \in (t, t')$, contradicting Lemma 9.

- If F_B were constant on (t, t'), then F_A and F_C would be strictly increasing there, so (IC:A) and (IC:C) would hold a.e. on (t, t'). This forces $\lambda_C = \lambda^{AG}$ a.e. while $\lambda_A > \lambda^{AG}$ a.e., implying $z_A(s) > z_C(s)$ for some $s \in (t, t')$, contradicting Lemma 7.

Hence all three F_A, F_B, F_C are strictly increasing on (t, t').

Step 2: all three indifference conditions hold locally and imply a contradiction. Because all three players are active on (t, t'), Lemma 4 implies that (IC:A)–(IC:C) hold a.e. on (t, t').

Define

$$\tau := \sup\Big\{ s \geq t : \ z_A(u) \geq z_C(u) > z_B(u) \text{ for all } u \in [t, s] \Big\},$$

By Lemma 7, the strict inequality $z_A > z_C$ cannot occur at any time $u > t$ once A is active, hence we must have $z_A(u) = z_C(u)$ for all $u \in (t, \tau)$.

Then $g_C^A(u) = g_C^B(u)$ on (t, τ), and (IC:A)–(IC:B) imply that $\lambda_C = \lambda^{AG}$ a.e. on (t, τ). Further, since $z_A(u) = z_C(u)$ for a.e. $u \in (t, \tau) \implies \lambda_A(u) = \lambda^{AG}$ for a.e. $u \in (t, \tau)$. Plugging into (IC:C) yields $\lambda_B < \lambda^{AG}$ a.e. on (t, τ) since $g_B((\cdot)) > 0$ on this interval. Therefore, for $u > t$ close enough to t, we have $z_A(u) = z_C(u) > z_B(u)$ and $z_B(u)$ is growing strictly more slowly, which implies that z_A and z_C reach 1 strictly before z_B can, contradicting Lemma 2.

□

Lemma 11. *Once all three players have conceded with positive probability, posteriors coincide: if $z_i(t) > z_i(0)$ for all $i \in N$, then*

$$z_A(t) = z_B(t) = z_C(t).$$

Proof. Fix $t > 0$ such that $z_i(t) > z_i(0)$ for all $i \in N$. If the three posteriors are not equal at t, then (up to swapping labels A and B) one of the following orderings must hold:

$$\begin{aligned}
&z_A(t) > z_C(t) \geq z_B(t),\\
&z_A(t) \geq z_B(t) > z_C(t),\\
&z_C(t) > \max\{z_A(t), z_B(t)\},\\
&z_A(t) = z_C(t) > z_B(t).
\end{aligned}$$

These are ruled out by Lemmas 7 – 10. Therefore $z_A(t) = z_B(t) = z_C(t)$.

□

Proof of Proposition A.1. **Step 1: Player C is active immediately after 0.** We claim $t_C = 0$. Suppose instead that $t_C > 0$, so that F_C is constant on $(0, t_C)$ and hence $\lambda_C(t) = 0$ for a.e. $t \in (0, t_C)$. By Lemma 3, at most one player's cdf can be constant on an interval before T, so both F_A and F_B must be strictly increasing on $(0, t_C)$, i.e. $\lambda_A(t), \lambda_B(t) > 0$ for a.e. $t \in (0, t_C)$. Then Lemma 4 implies that the local indifference conditions for A and B hold a.e. on $(0, t_C)$. Since $\lambda_C = 0$ there, those indifference conditions force $g_C^A(t) > 0$ and $g_C^B(t) > 0$ a.e. on $(0, t_C)$, hence $z_A(t) > z_C(t)$ and $z_B(t) > z_C(t)$ on a set of positive measure. This contradicts Lemma 8. Therefore $t_C = 0$.

Step 2: Once all three players are active, posteriors (and hazard rates) coincide. Let $t^* := \max\{t_A, t_B, t_C\}$ be the time at which the last player becomes active. By Lemma 11, we have

$$z_A(t) = z_B(t) = z_C(t) \qquad \text{for all } t \in [t^*, T).$$

Plugging this equality into the local indifference conditions, together with

$$z_i(t) = z_i(t^*) \exp\left(\int_{t^*}^{t} \lambda_i(s) ds \right),$$

implies that all three players' hazard rates coincide a.e. on (t^*, T), and hence must equal the AG hazard rate

$$\lambda_A(t) = \lambda_B(t) = \lambda_C(t) = \lambda^{AG} := \frac{r(1-\alpha)}{2\alpha - 1} \qquad \text{for a.e. } t \in (t^*, T).$$

We now pin down the *time-0 atoms* and the *identity of the initially inactive player.*

Case 1: $z_C(0) > \max\{z_A(0), z_B(0)\}$**.** Since $t_C = 0$ (Step 1), C concedes with positive density immediately after 0. If neither A nor B concedes with an atom at 0, then for all sufficiently small $t > 0$ we have $z_C(t) > z_A(t)$ and $z_C(t) > z_B(t)$, contradicting Lemma 9. Thus at least one of A, B must have an atom at 0.

We claim both must. Suppose wlog that only A has an atom at 0, so $F_A(0) > 0$ and $F_B(0) = 0$. Then $z_A(0+) \geq z_C(0) > z_B(0)$. If $z_A(0+) > z_C(0)$, then by continuity of posteriors we would have $z_A(t) > z_C(t) > z_B(t)$ for all small $t > 0$, contradicting Lemma 7. If instead $z_A(0+) = z_C(0) > z_B(0)$, then for small $t > 0$ we obtain the forbidden configuration $z_A(t) = z_C(t) > z_B(t)$, contradicting Lemma 10. Hence it is impossible that only one of A and B has an atom at 0.

Therefore both A and B concede with atoms at 0, and those atoms are uniquely pinned down by

$$z_A(0+) = z_B(0+) = z_C(0) \quad \Longleftrightarrow \quad \frac{z_A(0)}{1 - F_A(0)} = \frac{z_B(0)}{1 - F_B(0)} = z_C(0).$$

After time 0, all three are active immediately and hence concede at rate λ^{AG} until T (Step 2).

Case 2: $z_A(0) = z_B(0) > z_C(0)$**.** By Step 1, C is active immediately after 0. If $F_C(0) = 0$, then $z_C(t) = z_C(0) < z_A(0) = z_B(0)$ at $t = 0$, and since A, B are at least as reputable as C, for all small $t > 0$ we obtain $\min\{z_A(t), z_B(t)\} > z_C(t)$, contradicting Lemma 8. Hence $F_C(0) > 0$.

Because A and B start with equal posteriors and Lemma 9 rules out $z_C(0+) > z_A(0)$, the time-0 atom of C must satisfy

$$z_C(0+) = z_A(0) = z_B(0), \qquad \text{i.e.} \qquad F_C(0) = 1 - \frac{z_C(0)}{z_A(0)}.$$

No atom by A or B is compatible with equilibrium. After time 0, all three are active and concede at rate λ^{AG} until T (Step 2).

Case 3: $z_A(0) = z_B(0) = z_C(0)$**.** Any time-0 atom by some player would create unequal posteriors at 0+, which is impossible by the ordering lemmas. Hence $F_A(0) = F_B(0) = F_C(0) = 0$ and all three start conceding immediately at the common hazard rate λ^{AG}.

Case 4: $z_A(0) > z_B(0) \geq z_C(0)$**.** We claim that A is the unique initially inactive player:

$$\lambda_A(t) = 0 \ \text{ on } [0, t_A) \quad \text{and} \quad \lambda_B(t), \lambda_C(t) > 0 \ \text{ on } (0, t_A),$$

for some $t_A > 0$. Indeed, by Step 1 we have $t_C = 0$. If A were active on an interval immediately after 0, then for small $t > 0$ we would have $z_A(t) > z_C(t)$ and $z_A(t) > z_B(t)$, contradicting Lemma 7. Thus A must be inactive initially; and by Lemma 3, both B and C must be active.

On the initial interval $(0, t_A)$, we therefore have $\lambda_A = 0$ and $\lambda_B, \lambda_C > 0$, so by Lemma 4 the indifference conditions for B and C hold a.e. on $(0, t_A)$. Since $\lambda_A = 0$, these conditions uniquely pin down the hazard rates on $(0, t_A)$ as

$$\lambda_C(t) = \lambda^{AG} \qquad \text{and} \qquad \lambda_B(t) = (1 + \pi_{AC})\lambda^{AG} \qquad \text{for a.e. } t \in (0, t_A).$$

By Step 2, the moment A becomes active we must have $z_A = z_B = z_C$. Since A is inactive up to t_A, we have $z_A(t) = z_A(0)$ for $t < t_A$, so the alignment condition at t_A is

$$z_B(t_A) = z_C(t_A) = z_A(0).$$

Using the hazard rates above, this system uniquely determines t_A and $z_C(0+)$ and hence $F_C(0)$. Equivalently,

$$z_A(0) = z_B(0) \exp\left((1+\pi_{AC})\lambda^{AG} t_A\right) = z_C(0+) \exp\left(\lambda^{AG} t_A\right), \quad \text{and} \quad z_C(0+) = \frac{z_C(0)}{1 - F_C(0)},$$

which yields

$$F_C(0) = 1 - \frac{z_C(0)}{z_A(0)^{\frac{\pi_{AC}}{1+\pi_{AC}}} \, z_B(0)^{\frac{1}{1+\pi_{AC}}}}.$$

After t_A, all three are active and concede at hazard rate λ^{AG} until T.

Case 5: $z_A(0) > z_C(0) > z_B(0)$**.** As in Case 4, A is initially inactive while B and C are active on an initial interval $(0, t_A)$, and the same indifference argument implies

$$\lambda_C(t) = \lambda^{AG}, \qquad \lambda_B(t) = (1 + \pi_{AC})\lambda^{AG} \qquad \text{for a.e. } t \in (0, t_A),$$

until posteriors align at t_A.

Let $\tilde{t}_C$ be the time it would take C to reach $z_A(0)$ under hazard rate λ^{AG} absent any atom, and let $\tilde{t}_B$ be the analogous time for B under hazard rate $(1+\pi_{AC})\lambda^{AG}$ absent any atom:

$$\tilde{t}_C := -\frac{1}{\lambda^{AG}}\log\Big(\frac{z_C(0)}{z_A(0)}\Big), \qquad \tilde{t}_B := -\frac{1}{(1+\pi_{AC})\lambda^{AG}}\log\Big(\frac{z_B(0)}{z_A(0)}\Big).$$

If $\tilde{t}_C > \tilde{t}_B$, then (absent atoms) C would reach $z_A(0)$ strictly later than B. Since only one opponent can concede with an atom at time 0 in the negotiation between B and C, C must be the one who has an atom at 0 to speed up her posterior (and B must have none). Hence,

$$F_C(0) = 1 - \frac{z_C(0)}{z_A(0)^{\frac{\pi_{AC}}{1+\pi_{AC}}}\, z_B(0)^{\frac{1}{1+\pi_{AC}}}}, \qquad F_B(0) = 0.$$

If instead $\tilde{t}_B > \tilde{t}_C$, then B must be the one who has an atom at 0 (and C must have none), yielding

$$F_B(0) = 1 - \frac{z_B(0)\, z_A(0)^{\pi_{AC}}}{z_C(0)^{1+\pi_{AC}}}, \qquad F_C(0) = 0.$$

After t_A, all three are active and concede at hazard rate λ^{AG} until T.

Uniqueness. In each case, the identity of the initially inactive player is pinned down by the posterior-ordering lemmas together with Lemma 3. Given the set of active players on any interval, the local indifference conditions uniquely determine the hazards on that interval. Finally, the size (and identity) of the time-0 atom(s) is uniquely pinned down by the requirement that posteriors coincide at the time t^* when the last player becomes active (Step 2), together with the fact that on any contested negotiation at most one player can place an atom at time 0. This proves that the equilibrium described in the proposition is the unique equilibrium.

□

A.3. Proof of Proposition 2

Proof. Fix the unique equilibrium σ^* characterized in Proposition A.1 (and let F^* denote the induced cdfs in the no-concession subgame). For each player $k \in \{A,B,C\}$, let $v_k^* := v_k(\sigma^*)$ denote the time-0 expected payoff of k's rational type.

Step 0 (Payoff representations and a useful limit). Recall $U_i(t;\sigma_{-i})$ for $i \in \{A,B\}$ and $U_C(t;\sigma_{-C})$ given in equations (1)–(2) in the main text. In particular, in equilibrium we may compute

$$v_i^* = U_i(t;\sigma_{-i}^*) \quad \text{for any } t \text{ in the support of } dF_i^* \text{ on } (0,\infty),$$

and analogously for C, by Lemma 4.

We will repeatedly use the following observation: if player k is active immediately after 0 (i.e. F^*_k is strictly increasing on $(0,\varepsilon)$ for some $\varepsilon > 0$), then

$$v^*_k = \lim_{t\downarrow 0} U_k(t;\sigma^*_{-k}).$$

This limit is obtained from (1)–(2) by retaining the time-0 contributions. Which terms matter depends on the case: for a peripheral, a time-0 atom by the other peripheral may contribute through the induced AG continuation term, while for C the limit combines the time-0 contributions from both negotiations. Accordingly, in the case-by-case comparisons below we evaluate the $t \downarrow 0$ limit directly from (1)–(2).

Step 1 (Benchmark payoffs in the bilateral AG game). In the bilateral AG benchmark over surplus π_{ij}, the unique equilibrium features a time-0 atom

$$F^{AG}_{ji}(0) \;=\; \max\Big\{1 - \frac{z_j(0)}{z_i(0)},\, 0\Big\}.$$

Player i's (time-0) equilibrium payoff in that bilateral game is therefore

$$v^{AG}_{ij}(z_i(0), z_j(0)) \;=\; \pi_{ij}\Big((1-\alpha) + (2\alpha - 1)\, F^{AG}_{ji}(0)\Big). \tag{6}$$

We write $v^{AG}_{iC} := v^{AG}_{iC}(z_i(0), z_C(0))$ and similarly for v^{AG}_{Ci}.

We now prove each payoff comparison in Proposition 2 case by case, using the equilibrium characterization in Proposition A.1.

Case 1: $z_C(0) \geq \max\{z_A(0), z_B(0)\}$. By Proposition A.1, C has no time-0 atom and each peripheral $i \in \{A,B\}$ (if weaker than C) concedes with an atom at 0 that raises $z_i(0+)$ to $z_C(0)$. Equivalently,

$$F^*_i(0) = \max\Big\{1 - \frac{z_i(0)}{z_C(0)},\, 0\Big\} = F^{AG}_{iC}(0), \qquad F^*_C(0) = 0.$$

After time 0, all posteriors are aligned and all players concede at the common AG hazard $\lambda^{AG} = \frac{r(1-\alpha)}{2\alpha-1}$, exactly as in the bilateral benchmark in each negotiation. Since continuation play after any concession is the unique AG continuation, the induced outcome distribution in each negotiation coincides with the AG benchmark, yielding

$$v^*_i = v^{AG}_{iC} \;\text{ for } i \in \{A,B\}, \qquad v^*_C = v^{AG}_{CA} + v^{AG}_{CB}.$$

Case 2: $z_C(0) < z_B(0) < z_A(0)$. Proposition A.1 implies: A is initially inactive, B and C are active immediately after 0, and C concedes with a time-0 atom $F^*_C(0) \in (0,1)$, while $F^*_A(0) = F^*_B(0) = 0$.

<u>C's payoff.</u> Since $F^*_C(0) > 0$, time 0 is in the support of C's concession distribution. Sequential rationality therefore implies that C is indifferent between conceding at 0 and following σ^*, hence

$$v^*_C = (1-\alpha)(\pi_{AC} + 1).$$

In the AG benchmark, C is the weaker party in both negotiations (since $z_C(0) < z_A(0)$ and $z_C(0) < z_B(0)$), so $v^{AG}_{C,A} = (1-\alpha)\pi_{AC}$ and $v^{AG}_{C,B} = (1-\alpha)\cdot 1$, implying $v^*_C = v^{AG}_{C,A} + v^{AG}_{C,B}$.

B's payoff. Player B is active immediately after 0, so $v^*_B = \lim_{t\downarrow 0} U_B(t; \sigma^*_{-B})$. Because $F^*_A(0) = 0$ and A is initially inactive, the only time-0 atom relevant for this limit is C's atom $F^*_C(0)$, yielding

$$v^*_B = \pi_{BC}\Big(\alpha F^*_C(0) + (1-\alpha)(1 - F^*_C(0))\Big) = \pi_{BC}\Big((1-\alpha) + (2\alpha - 1)F^*_C(0)\Big).$$

In the bilateral AG benchmark between B and C, C is the weak party, so the AG atom against B is $F^{AG}_{CB}(0) = 1 - \frac{z_C(0)}{z_B(0)}$ and

$$v^{AG}_{B,C} = \pi_{BC}\Big((1-\alpha) + (2\alpha - 1)F^{AG}_{CB}(0)\Big).$$

Finally, Proposition A.1 gives

$$F^*_C(0) = 1 - \frac{z_C(0)}{z_A(0)^{\frac{\pi_{AC}}{1+\pi_{AC}}} z_B(0)^{\frac{1}{1+\pi_{AC}}}} \quad \Rightarrow \quad F^*_C(0) > 1 - \frac{z_C(0)}{z_B(0)} = F^{AG}_{CB}(0),$$

because $z_A(0)^{\frac{\pi_{AC}}{1+\pi_{AC}}} z_B(0)^{\frac{1}{1+\pi_{AC}}} > z_B(0)$ when $z_A(0) > z_B(0)$. Therefore $v^*_B > v^{AG}_{BC}$.

A's payoff. In the AG benchmark between A and C, C concedes with a time-0 atom of size $F^{AG}_{CA}(0) = 1 - \frac{z_C(0)}{z_A(0)}$, which raises A's payoff above $(1-\alpha)\pi_{AC}$ via (6). In the three-player equilibrium, A is initially inactive and posteriors align only at the strictly positive time $t_A > 0$ at which A becomes active. Hence part of the probability mass of "C concedes before A concedes" that is realized at calendar time 0 in the bilateral benchmark is shifted to calendar times in $(0, t_A]$ in the three-player equilibrium. Because $\alpha > (1-\alpha)$ and discounting is strict, shifting any positive probability mass of receiving a concession from time 0 to a strictly positive time strictly lowers the time-0 expected payoff. Therefore $v^*_A < v^{AG}_{A,C}$.

Combining the three comparisons yields the claim in Case 2.

Case 3: $z_B(0) < z_C(0) < z_A(0)$. Proposition A.1 implies A is initially inactive and B and C are active immediately after 0. Moreover, exactly one of the following holds: either (a) C concedes with a time-0 atom $F^*_C(0) > 0$ and $F^*_B(0) = 0$, or (b) B concedes with a time-0 atom $F^*_B(0) > 0$ and $F^*_C(0) = 0$.

A's payoff. In both subcases, C is weaker than A initially and the bilateral benchmark for A and C features a strictly positive time-0 atom from C to A of size $1 - \frac{z_C(0)}{z_A(0)}$. In the three-player equilibrium, A remains inactive for an initial interval and posteriors align only at a strictly positive time $t_A > 0$. Hence, relative to the bilateral benchmark, some probability mass of being conceded to by C is shifted away from time 0 to later calendar times, which strictly reduces the discounted value. Therefore $v^*_A < v^{AG}_{AC}$.

B's payoff. If (a) $F^*_C(0) > 0$, then B is active immediately after 0 and the limit argument from Case 2 gives

$$v^*_B = \pi_{BC}\Big((1-\alpha) + (2\alpha - 1)F^*_C(0)\Big) > (1-\alpha)\pi_{BC}.$$

Since $z_B(0) < z_C(0)$, B is the weak player in the bilateral benchmark on BC, so $v^{AG}_{B,C} = (1-\alpha)\pi_{BC}$. Hence $v^*_B > v^{AG}_{B,C}$. If instead (b) $F^*_B(0) > 0$, then time 0 is in the support of B's concession distribution, so sequential rationality implies $v^*_B = (1-\alpha)\pi_{BC} = v^{AG}_{B,C}$. Thus, in either subcase, $v^*_B \geq v^{AG}_{B,C}$.

C's payoff. In the bilateral benchmark, C is weak against A but strong against B, so

$$v^{AG}_{C,A} + v^{AG}_{C,B} = (1-\alpha)(\pi_{AC}+1) + (2\alpha-1)\Big(1 - \frac{z_B(0)}{z_C(0)}\Big),$$

where the last (strictly positive) term is the strong-player premium in the negotiation between B and C.

If (a) $F^*_C(0) > 0$, then C concedes at time 0 with positive probability, hence $v^*_C = (1-\alpha)(\pi_{AC}+1)$ as in Case 2. This is strictly smaller than $v^{AG}_{C,A} + v^{AG}_{C,B}$ because $z_B(0) < z_C(0)$ implies $1 - \frac{z_B(0)}{z_C(0)} > 0$.

If (b) $F^*_B(0) > 0$ and $F^*_C(0) = 0$, then C is active immediately after 0 and we may evaluate $v^*_C = \lim_{t\downarrow 0} U_C(t;\sigma^*_{-C})$. In this limit, the only time-0 atom is B's concession at 0, and the continuation between A and C following B's concession is the bilateral AG game between A and C starting from $(z_A(0), z_C(0))$, in which C is the weak player and hence earns $(1-\alpha)\pi_{AC}$. Therefore,

$$v^*_C = (1-\alpha)(\pi_{AC}+1) + (2\alpha-1)F^*_B(0).$$

Moreover, Proposition A.1 yields

$$F_B(0) = 1 - \frac{z_B(0)\, z_A(0)^{\pi_{AC}}}{z_C(0)^{1+\pi_{AC}}} \;<\; 1 - \frac{z_B(0)}{z_C(0)} = F^{AG}_{BC}(0),$$

since $z_A(0)^{\pi_{AC}} > z_C(0)^{\pi_{AC}}$ when $z_A(0) > z_C(0)$. Hence $v^*_C < v^{AG}_{CA} + v^{AG}_{CB}$ also in subcase (b).

This completes the proof of Case 3 and hence of Proposition 2.

□

A.4. Proof of Proposition 3

Proof. Set $\pi_{AC} = \pi_{BC} = 1$ and write

$$z^\varepsilon_B(0) = \varepsilon, \qquad z^\varepsilon_C(0) = \kappa_B\varepsilon, \qquad z^\varepsilon_A(0) = \kappa_A\kappa_B\varepsilon,$$

with $\kappa_A, \kappa_B > 1$ fixed. To lighten notation, suppress the dependence on ε.

In the bilateral AG benchmark,

$$F^{AG}_{CA}(0) = 1 - \frac{z_C(0)}{z_A(0)} = 1 - \frac{1}{\kappa_A}, \qquad F^{AG}_{BC}(0) = 1 - \frac{z_B(0)}{z_C(0)} = 1 - \frac{1}{\kappa_B}.$$

Hence

$$v_{CA}^{AG} = 1-\alpha, \qquad v_{CB}^{AG} = \alpha - \frac{2\alpha-1}{\kappa_B}, \qquad v_{AC}^{AG} = \alpha - \frac{2\alpha-1}{\kappa_A}, \qquad v_{BC}^{AG} = 1-\alpha.$$

We now consider the two cases identified in Proposition A.1.

Case 1: $\kappa_A > \kappa_B$. By Proposition A.1, player C concedes at $t=0$ with atom

$$F_C^*(0) = 1 - \sqrt{\frac{\kappa_B}{\kappa_A}}, \qquad F_B^*(0) = 0.$$

Since C is indifferent at $t=0$,

$$v_C^* = 2(1-\alpha).$$

Therefore

$$v_{CA}^{AG} + v_{CB}^{AG} - v_C^* = (1-\alpha) + \left(\alpha - \frac{2\alpha-1}{\kappa_B}\right) - 2(1-\alpha) = (2\alpha-1)\left(1-\frac{1}{\kappa_B}\right).$$

Moreover, B is active immediately after 0, so

$$v_B^* = \alpha F_C^*(0) + (1-\alpha)\big(1-F_C^*(0)\big) = \alpha - (2\alpha-1)\sqrt{\frac{\kappa_B}{\kappa_A}} > 1-\alpha = v_{BC}^{AG}.$$

Finally, part (ii) follows directly from Proposition 2: for every prior vector satisfying $z_A(0) > z_C(0) > z_B(0)$, we have

$$v_A^* < v_{AC}^{AG}.$$

Case 2: $\kappa_A \le \kappa_B$. By Proposition A.1, player B concedes at $t=0$ with atom

$$F_B^*(0) = 1 - \frac{\kappa_A}{\kappa_B}, \qquad F_C^*(0) = 0.$$

Because B is indifferent at $t=0$,

$$v_B^* = 1-\alpha = v_{BC}^{AG}.$$

When B concedes at $t=0$, player C receives α from the B–C negotiation immediately and $(1-\alpha)$ from the continuation against A. When B does not concede at $t=0$, player C earns $2(1-\alpha)$ by indifference. Hence

$$v_C^* = 2(1-\alpha) + (2\alpha-1)\left(1-\frac{\kappa_A}{\kappa_B}\right).$$

Therefore

$$v_{CA}^{AG} + v_{CB}^{AG} - v_C^* = (2\alpha-1)\left(1-\frac{1}{\kappa_B}\right) - (2\alpha-1)\left(1-\frac{\kappa_A}{\kappa_B}\right) = (2\alpha-1)\frac{\kappa_A-1}{\kappa_B}.$$

Again, part (ii) is immediate from Proposition 2.

Combining the two cases yields

$$v_{CA}^{AG} + v_{CB}^{AG} - v_C^* = (2\alpha-1)\frac{\min\{\kappa_A,\kappa_B\}-1}{\kappa_B} > 0,$$

and

$$v_B^* \ge v_{BC}^{AG},$$

with strict inequality if and only if $\kappa_A > \kappa_B$. Since the expressions above do not depend on ε, the same formulas describe the limit as $\varepsilon \downarrow 0$. □

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Online Appendix

B. Sequential Negotiations

In this section, we relax the assumption of simultaneous negotiations. To this end, consider an environment in which players A and C bargain first over surplus $\pi_{AC} > 0$ according to the same continuous-time war-of-attrition protocol as in the baseline, and only after the first-stage dispute ends do players B and C bargain over surplus $\pi_{BC} = 1$. All concessions and agreement times are publicly observed. Types are as in the baseline: each player $i \in \{A, B, C\}$ is behavioral with prior $z_i(0) \in (0,1)$ (never concedes) and rational otherwise. Player C has a single global type. Histories and strategies are adapted in the obvious way. Payoffs are additive across negotiations and discounted (in calendar time).

Let t denote the calendar time at which the first-stage dispute (between A and C) ends. If A concedes at t, then C's type is not revealed; the second-stage game between B and C is a bilateral AG reputational war of attrition with initial reputations $(z_B(0), z_C(t))$, where $z_C(t)$ is the posterior that C is behavioral after observing no concession by C up to time t. If instead C concedes to A at t, then C is revealed rational at t^+, and hence in the second-stage AG continuation with initial reputations $(z_B(0), 0)$ the (unique) equilibrium has C conceding immediately with probability 1.

Formally, define (as before) $V_{CB}^{AG}(z_C, z_B)$ as C's time-0 equilibrium payoff in a bilateral AG game over surplus 1 against B with reputations (z_C, z_B):

$$V_{CB}^{AG}(z_C, z_B) = (1-\alpha) + (2\alpha - 1)\max\left\{1 - \frac{z_B}{z_C}, 0\right\}.$$

(Equivalently, B concedes at the start of stage 2 with probability $\max\{1 - z_B/z_C, 0\}$.)

As Proposition B.1 shows, there exists a unique equilibrium in this environment. The first-stage bargaining between A and C features a common terminal time T at which posteriors reach 1. However, the presence of the second-stage negotiation fundamentally alters first-stage incentives. In particular, the central player's continuation value from a future negotiation between B and C enters the local indifference condition of the first-stage war of attrition. Specifically, player C is indifferent at time t if

$$r(\pi_{AC} + 1)(1-\alpha) = \left(\alpha\pi_{AC} + V_{CB}^{AG}(z_C(t), z_B(0)) - (\pi_{AC} + 1)(1-\alpha)\right)\frac{f_A(t)}{1 - F_A(t)}.$$

As a result, equilibrium concession hazard rates in the first stage are asymmetric even when initial reputations are symmetric. Specifically, for $t \in (0, T)$,

$$\lambda_C(t) = \lambda^{AG}, \text{ and } \lambda_A(t) = \frac{(\pi_{AC} + 1)}{\left(\pi_{AC} + \max\left\{1 - \frac{z_B(0)}{z_C(t)}, 0\right\}\right)}\lambda^{AG} \geq \lambda^{AG}. \tag{7}$$

Because A concedes at a higher rate, A's posterior reputation rises faster than C's. To reconcile this with the common-terminal-time requirement, equilibrium may require an atom of concession by C at time 0, even if $z_A(0) < z_C(0)$.

Two implications follow. First, even when initial reputations are symmetric, the central player is strictly worse off than in the bilateral benchmark, while A is strictly better off. Second, this disadvantage may persist when C is stronger than A: the faster posterior growth of A may still force an initial adjustment by C. Once the first-stage negotiation ends, the second stage bargaining between B and C reduces to a standard bilateral war of attrition with beliefs updated from the first-stage history. In particular, if A concedes, the continuation game between B and C is exactly the bilateral benchmark; if C concedes, she is revealed to be rational and concedes immediately in the second stage. Let us now take a closer look at each stage.

Stage 2 (benchmark continuation). Given any stage 2 start time $\tau \geq 0$, the continuation game between B and C is the bilateral AG reputational war of attrition with priors $(z_B(0), \widehat{z}_C)$, where $\widehat{z}_C$ is the public posterior on C at the start of stage 2. Since $\pi_{BC} = 1$, we denote by $V_{CB}^{AG}(\widehat{z}_C, z_B(0))$ the (current-value) equilibrium payoff of a *rational* C in this bilateral game. Using the benchmark characterization recalled in Section 3,

$$V_{CB}^{AG}(z_C, z_B) = (1-\alpha) + (2\alpha - 1)\, g(z_C; z_B), \qquad g(z_C; z_B) := \max\Big\{1 - \frac{z_B}{z_C}, 0\Big\}. \tag{8}$$

Here $g(z_C; z_B)$ is exactly the equilibrium time-0 concession probability of B in the bilateral B–C game when $z_C > z_B$ (and equals 0 when $z_C \leq z_B$).

Stage 1 (payoffs as functions of the stage-2 continuation). Fix any PBE. Consider stage 1 at time t along the stage-1 no-concession history. If A concedes at time t, then the stage 1 negotiation ends with C receiving $\alpha\pi_{AC}$ and A receiving $(1-\alpha)\pi_{AC}$, and then stage 2 begins immediately with public posterior $\widehat{z}_C = z_C(t)$ (because C has not conceded up to t). Thus the *current-value* payoff to a rational C at time t from an A-concession equals

$$W_C(t) := \alpha\pi_{AC} + V_{CB}^{AG}(z_C(t), z_B(0)). \tag{9}$$

If instead C concedes at time t in stage 1, then she is revealed rational; in stage 2 she concedes immediately to B (a rational C strictly prefers immediate concession against a known-rational type than any delay, and B never concedes against a player known to concede immediately). Therefore the current-value payoff to a rational C from conceding at time t in stage 1 is

$$(1-\alpha)\pi_{AC} + (1-\alpha)\cdot 1 = (1-\alpha)(\pi_{AC} + 1). \tag{10}$$

Proposition B.1 (Sequential negotiations: unique equilibrium)**.** *There exists a unique PBE of the sequential game. In this PBE:*

1. *(**Stage 2**) If stage 1 ends at time τ because A concedes, then stage 2 is the bilateral AG equilibrium between B and C with initial posteriors $(z_B(0), z_C(\tau))$. If stage 1 ends at time τ because C concedes, then C is revealed rational and concedes immediately to B at the start of stage 2.*

2. *(**Stage 1 hazards**) Along the stage-1 no-concession history, there is a finite terminal time $T \in (0, \infty)$ such that $z_A(T) = z_C(T) = 1$. On $(0, T)$ both players concede with*

positive density a.e., and their equilibrium hazards satisfy

$$\lambda_C(t) = \lambda^{AG} := \frac{r(1-\alpha)}{2\alpha - 1}, \qquad \lambda_A(t) = \lambda^{AG} \cdot \frac{\pi_{AC}+1}{\pi_{AC} + g(z_C(t); z_B(0))} \quad \textit{for a.e. } t \in (0,T), \tag{11}$$

where $g(\cdot;\cdot)$ is defined in (8).

Equivalently, if $z_C(t) \le z_B(0)$ then $\lambda_A(t) = \lambda^{AG} \cdot \frac{\pi_{AC}+1}{\pi_{AC}}$, while if $z_C(t) > z_B(0)$ then $\lambda_A(t) = \lambda^{AG} \cdot \frac{\pi_{AC}+1}{\pi_{AC}+1-z_B(0)/z_C(t)}$.

3. *(**Time-0 atom and terminal time**) At most one player concedes with positive probability at time 0 in stage 1. Let $z_i(0+) := z_i(0)/(1-F_i(0))$ denote the posteriors after any time-0 atom in stage 1. The terminal time is*

$$T = -\frac{1}{\lambda^{AG}} \log z_C(0+). \tag{12}$$

Define the threshold $\bar{z}_A$ by

$$\bar{z}_A := \begin{cases} \dfrac{(\pi_{AC}+1)\, z_C(0) - z_B(0)}{\pi_{AC}+1-z_B(0)}, & \textit{if } z_C(0) \ge z_B(0), \\[2ex] \dfrac{\pi_{AC}\, z_B(0)}{\pi_{AC}+1-z_B(0)} \left(\dfrac{z_C(0)}{z_B(0)}\right)^{\frac{\pi_{AC}+1}{\pi_{AC}}}, & \textit{if } z_C(0) < z_B(0). \end{cases}$$

(i) *If $z_A(0) < \bar{z}_A$, then A bears the unique time-0 atom and C does not:*

$$F_A(0) = 1 - \frac{z_A(0)}{\bar{z}_A}, \qquad F_C(0) = 0, \qquad z_C(0+) = z_C(0), \qquad z_A(0+) = \bar{z}_A.$$

(ii) *If $z_A(0) = \bar{z}_A$, then there is no time-0 atom:*

$$F_A(0) = F_C(0) = 0, \qquad z_A(0+) = z_A(0), \qquad z_C(0+) = z_C(0).$$

(iii) *If $z_A(0) > \bar{z}_A$, then C bears the unique time-0 atom and A does not:*

$$F_A(0) = 0, \qquad F_C(0) = 1 - \frac{z_C(0)}{z_C(0+)}, \qquad z_A(0+) = z_A(0),$$

where $z_C(0+)$ is uniquely determined as follows. Let

$$\tilde{z}_A := \frac{\pi_{AC}\, z_B(0)}{\pi_{AC}+1-z_B(0)}.$$

If $z_A(0) \ge \tilde{z}_A$, then $z_C(0+) \ge z_B(0)$ and

$$z_C(0+) = \frac{z_B(0) + z_A(0)\,(\pi_{AC}+1-z_B(0))}{\pi_{AC}+1}.$$

If $z_A(0) < \tilde{z}_A$, then $z_C(0+) < z_B(0)$ and

$$z_C(0+) = z_B(0) \left(\frac{z_A(0)\,(\pi_{AC}+1-z_B(0))}{\pi_{AC}\, z_B(0)} \right)^{\frac{\pi_{AC}}{\pi_{AC}+1}}.$$

Sketch of proof. **Step 1 (Stage 2 is pinned down).** Fix any public history at the start of stage 2. Conditional on this history, the continuation game is exactly a bilateral reputational war of attrition with one commitment type for each player. By the bilateral benchmark (AG), the equilibrium is unique and yields the current-value payoff $V_{CB}^{AG}(\cdot,\cdot)$ for a rational C, given by (8). This proves part 1 of the proposition.

Step 2 (No stalling in stage 1). Consider stage 1 and any interval $(t_0, t_1) \subset (0, \infty)$ along the stage-1 no-concession history. If $\lambda_A(t) = \lambda_C(t) = 0$ for a.e. $t \in (t_0, t_1)$, then conditional on reaching t_0 the negotiation cannot end on (t_0, t_1). Since discounting is strict, any rational player strictly prefers conceding at t_0 to conceding at any later time in $(t_0, t_1]$, contradicting sequential rationality. Hence it cannot be that both hazards are zero a.e. on any positive-length interval.

Step 3 (Local indifference for A pins down λ_C). Fix $t \in (0, T)$ such that A concedes with positive density at t (equivalently, $\lambda_A(t) > 0$) and $F_A(t) < 1$. Then following standard arguments,

$$r(1-\alpha)\pi_{AC} = (2\alpha - 1)\pi_{AC}\lambda_C(t).$$

Hence, $\lambda_C(t) = \lambda^{AG}$ for a.e. $t \in (0, T)$.

Step 4 (Local indifference for C pins down $\lambda_A(t)$). Fix $t \in (0, T)$ such that C concedes with positive density at t (so $\lambda_C(t) > 0$) and $F_C(t) < 1$. Indifference of C at time t implies

$$r(1-\alpha)(\pi_{AC} + 1) = \lambda_A(t)\big(W_C(t) - (1-\alpha)(\pi_{AC} + 1)\big). \tag{13}$$

Substituting $W_C(t) = \alpha\pi_{AC} + V_{CB}^{AG}(z_C(t), z_B(0))$ from (9) and using (8), and $\lambda^{AG} = \frac{r(1-\alpha)}{2\alpha-1}$, and plugging into (13) yields

$$\lambda_A(t) = \frac{r(1-\alpha)(\pi_{AC} + 1)}{(2\alpha - 1)\big(\pi_{AC} + g(z_C(t); z_B(0))\big)} = \lambda^{AG} \cdot \frac{\pi_{AC} + 1}{\pi_{AC} + g(z_C(t); z_B(0))}.$$

This proves (11).

Step 5 (Posteriors and existence of a finite terminal time). Since $\lambda_C(t) = \lambda^{AG} > 0$ a.e. on $(0, T)$, Bayes' rule implies $z_C(t) = z_C(0+)\exp(\lambda^{AG} t)$ along the stage-1 no-concession history until the strategy support ends. Because $z_C(0+) \in (0, 1)$, there is a unique finite time T at which $z_C(T) = 1$, namely (12). At this time, $F_C(T) = 1 - z_C(0)$, so a rational C has conceded in stage 1 with probability 1 by time T.

Next we show that necessarily $z_A(T) = 1$ as well. Suppose instead that $z_A(T) < 1$. Then conditional on no concession up to time T, there is positive probability that A is rational. But at time T, we have $z_C(T) = 1$, so conditional on the same history player C is behavioral with probability one, hence will never concede. Therefore any rational A who has not conceded by T strictly prefers conceding immediately at T (yielding $(1-\alpha)\pi_{AC}$) to any strategy that delays concession beyond T (which can only weakly reduce his discounted payoff because it cannot induce concession by C). This contradicts sequential rationality unless a rational A concedes by time T with probability one, which is equivalent to $z_A(T) = 1$. Hence along the stage-1 no-concession history we must have $z_A(T) = z_C(T) = 1$, proving the terminal-time claim in part 2.

Step 6 (Solving for the time-0 atom). Because $z_i(0+) = z_i(0)/(1-F_i(0))$, any time-0 atom by player i weakly increases $z_i(0+)$. We first note that at most one player concedes with positive probability at time 0: if both A and C placed positive mass at 0, then conditional on being rational each would strictly prefer shifting an ε-amount of mass from 0 to a very small $\delta > 0$ to avoid the strictly worse simultaneous-concession outcome at 0, contradicting optimality. Hence at most one of $F_A(0), F_C(0)$ is positive.

We now compute the boundary condition $z_A(T) = 1$ explicitly as a function of $(z_A(0+), z_C(0+))$. Recall $z_C(t) = z_C(0+)\exp(\lambda^{AG}t)$.

Case 6.1: $z_C(0+) \geq z_B(0)$. Then for all $t \in [0,T]$ we have $z_C(t) \geq z_B(0)$, so $g(z_C(t); z_B(0)) = 1 - z_B(0)/z_C(t)$. From (11) we obtain the ODE

$$\frac{\dot{z}_A(t)}{z_A(t)} = \lambda_A(t) = \lambda^{AG} \cdot \frac{\pi_{AC}+1}{\pi_{AC}+1-z_B(0)/z_C(t)}.$$

Using $\dot{z}_C(t) = \lambda^{AG} z_C(t)$ and the change of variables z_A as a function of z_C, one verifies that

$$\frac{d}{dt}\log\Big((\pi_{AC}+1)z_C(t) - z_B(0)\Big) = \lambda_A(t),$$

hence integrating from 0 to t yields

$$z_A(t) = z_A(0+) \cdot \frac{(\pi_{AC}+1)z_C(t) - z_B(0)}{(\pi_{AC}+1)z_C(0+) - z_B(0)}.$$

At $t = T$ we have $z_C(T) = 1$, so the boundary condition $z_A(T) = 1$ becomes

$$1 = z_A(0+) \cdot \frac{\pi_{AC}+1-z_B(0)}{(\pi_{AC}+1)z_C(0+) - z_B(0)}. \tag{14}$$

Case 6.2: $z_C(0+) < z_B(0)$. Let $t_B := \frac{1}{\lambda^{AG}}\log\frac{z_B(0)}{z_C(0+)} \in (0,T)$ so that $z_C(t_B) = z_B(0)$. For $t \in [0, t_B]$ we have $g(z_C(t); z_B(0)) = 0$, so $\lambda_A(t) = \lambda^{AG}\frac{\pi_{AC}+1}{\pi_{AC}}$ is constant and thus

$$z_A(t_B) = z_A(0+)\exp\left(\lambda^{AG}\frac{\pi_{AC}+1}{\pi_{AC}}t_B\right) = z_A(0+)\left(\frac{z_B(0)}{z_C(0+)}\right)^{\frac{\pi_{AC}+1}{\pi_{AC}}}.$$

For $t \in [t_B, T]$ we have $z_C(t) \geq z_B(0)$, hence the linear formula from Case 6.1 applies starting at t_B:

$$z_A(t) = z_A(t_B) \cdot \frac{(\pi_{AC}+1)z_C(t) - z_B(0)}{(\pi_{AC}+1)z_C(t_B) - z_B(0)} = z_A(t_B) \cdot \frac{(\pi_{AC}+1)z_C(t) - z_B(0)}{\pi_{AC} z_B(0)}.$$

Evaluating at $t = T$ where $z_C(T) = 1$, the boundary condition $z_A(T) = 1$ becomes

$$1 = z_A(0+)\left(\frac{z_B(0)}{z_C(0+)}\right)^{\frac{\pi_{AC}+1}{\pi_{AC}}} \cdot \frac{\pi_{AC}+1-z_B(0)}{\pi_{AC} z_B(0)}. \tag{15}$$

Step 7 (Uniqueness and the explicit atoms). Clearly, there can be at most one atom at 0.

(a) C has no atom: $z_C(0+) = z_C(0)$. If $z_C(0) \geq z_B(0)$, then (14) implies the unique required value of $z_A(0+)$ is

$$z_A(0+) = \frac{(\pi_{AC}+1)z_C(0) - z_B(0)}{\pi_{AC}+1-z_B(0)} =: \bar{z}_A.$$

If $z_C(0) < z_B(0)$, then (15) implies the unique required value of $z_A(0+)$ is

$$z_A(0+) = \frac{\pi_{AC} z_B(0)}{\pi_{AC}+1-z_B(0)} \left(\frac{z_C(0)}{z_B(0)}\right)^{\frac{\pi_{AC}+1}{\pi_{AC}}} =: \bar{z}_A.$$

In either subcase, if $z_A(0) < \bar{z}_A$ then the only way to achieve $z_A(0+) = \bar{z}_A$ is that A places a time-0 atom of size $F_A(0) = 1 - z_A(0)/\bar{z}_A$, while $F_C(0) = 0$. If $z_A(0) = \bar{z}_A$, no atom is needed. If $z_A(0) > \bar{z}_A$, then A cannot reduce $z_A(0+)$ (atoms only increase posteriors), so C must instead have the unique time-0 atom.

(b) A has no atom: $z_A(0+) = z_A(0)$. In this case C must choose $z_C(0+) \geq z_C(0)$ such that the boundary condition holds. If we seek a solution with $z_C(0+) \geq z_B(0)$, then (14) uniquely yields

$$(\pi_{AC}+1)z_C(0+) - z_B(0) = z_A(0)\,(\pi_{AC}+1-z_B(0)), \quad \text{so} \quad z_C(0+) = \frac{z_B(0) + z_A(0)(\pi_{AC}+1-z_B(0))}{\pi_{AC}+1}.$$

This candidate indeed satisfies $z_C(0+) \geq z_B(0)$ if and only if $z_A(0) \geq \tilde{z}_A := \frac{\pi_{AC} z_B(0)}{\pi_{AC}+1-z_B(0)}$. If instead we seek a solution with $z_C(0+) < z_B(0)$, then (15) uniquely yields

$$z_C(0+) = z_B(0) \left(\frac{z_A(0)(\pi_{AC}+1-z_B(0))}{\pi_{AC} z_B(0)} \right)^{\frac{\pi_{AC}}{\pi_{AC}+1}},$$

and this candidate satisfies $z_C(0+) < z_B(0)$ if and only if $z_A(0) < \tilde{z}_A$. In either subcase C's time-0 atom size is uniquely $F_C(0) = 1 - z_C(0)/z_C(0+)$ and $F_A(0) = 0$.

Combining parts (a) and (b) yields the case distinction and formulas stated in part 3. Given $(z_A(0+), z_C(0+))$, the hazards (11) pin down the full stage-1 concession-time distributions (via $\dot{z}_i(t) = \lambda_i(t) z_i(t)$ and Bayes' rule), and stage 2 play is uniquely pinned down by Step 1. Therefore, the PBE is unique. □

C. Partial observability of concessions

We relax the assumption that the identity of the conceding player is publicly observed. Suppose that when the negotiation between $i \in \{A, B\}$ and C ends at time t, the uninvolved peripheral k observes only that an agreement was reached at time t, not which party conceded. We restrict attention to the symmetric case $z_A(0) = z_B(0) = z_C(0) =: z_0$ and $\pi_{AC} = \pi_{BC} = 1$.

Until the first agreement, public histories coincide with those in the baseline, so posteriors are common along the no-concession path. After an agreement in the other negotiation, the uninvolved peripheral's belief about C need not coincide with the involved players' beliefs. Accordingly, write $z_C^k(t)$ for peripheral k's posterior probability at time t that C is behavioral.

Belief updating. Along the no-concession path, posteriors are common. When the i–C negotiation ends at time t, peripheral k's posterior about C jumps to

$$z_C^k(t^+) \;=\; z_C(t^-)\,\frac{\lambda_i(t)}{\lambda_i(t)+\lambda_C(t)}, \tag{16}$$

which is a strict downward revision whenever $\lambda_C(t) > 0$. In the unique AG continuation of the k–C negotiation, C then concedes immediately at t^+ with atom

$$g_C^k(t) \;=\; \max\left\{1 - \frac{z_C^k(t^+)}{z_k(t^+)},\, 0\right\}.$$

Candidate equilibrium. We conjecture that A and B concede at rate λ^{AG} throughout, while C concedes with a strictly positive atom $F_C(0) > 0$ at $t = 0$ and then at a time-varying rate $\lambda_C(t)$ on $(0, T)$. Under this profile,

$$z_A(t) = z_B(t) = z_0 e^{\lambda^{AG} t}, \qquad z_C(t) = z_C(0^+)\exp\!\Big(\int_0^t \lambda_C(s)\,ds\Big),$$

where $z_C(0^+) = z_0/(1 - F_C(0))$.

***A*'s indifference condition pins down λ_C.** Under (16) with $\lambda_i = \lambda^{AG}$, denote A's updated belief about C upon observing the B–C agreement at time t by

$$\hat{z}_C(t) \;:=\; z_C^A(t^+) \;=\; z_C(t)\,\frac{\lambda^{AG}}{\lambda^{AG}+\lambda_C(t)}.$$

The atom that C makes to A in the AG continuation is then

$$g_C^A(t) \;=\; \max\left\{1 - \frac{\hat{z}_C(t)}{z_A(t)}, 0\right\} \;=\; 1 - \frac{z_C(t)}{z_A(t)} \cdot \frac{\lambda^{AG}}{\lambda^{AG}+\lambda_C(t)}.$$

For A to be indifferent, $\lambda_C(t) + \lambda^{AG} \cdot g_C^A(t) = \lambda^{AG}$, which simplifies to

$$\lambda_C(t)\big(\lambda^{AG}+\lambda_C(t)\big) \;=\; \big(\lambda^{AG}\big)^2\,\frac{z_C(t)}{z_A(t)}. \tag{17}$$

This quadratic has unique positive root

$$\lambda_C(t) \;=\; \frac{\lambda^{AG}}{2}\left(-1 + \sqrt{1 + 4\,\frac{z_C(t)}{z_A(t)}}\right), \tag{18}$$

and by symmetry B's indifference condition yields the same expression.

Proposition B.2 (Partial observability)**.** *Suppose $z_A(0) = z_B(0) = z_C(0) = z_0$, $\pi_{AC} = \pi_{BC} = 1$, and concessions are partially observable. There exists an equilibrium in which A and B concede at rate λ^{AG} throughout and C concedes with atom $F_C(0) = 1 - 1/h(0) \in (0, 1)$ at $t = 0$*

and thereafter at rate $\lambda_C(t) < \lambda^{AG}$ given by (18), where $h(0) > 1$ is uniquely determined. Equilibrium payoffs satisfy

$$v_A^* = v_B^* = (1-\alpha) + (2\alpha - 1)F_C(0) > 1 - \alpha = v_A^{AG}, \qquad v_C^* = 2(1-\alpha) = v_C^{AG}.$$

Thus A and B are strictly better off than in the bilateral benchmark while C's payoff is unchanged.

Sketch of Proof. Step 1: A's and B's indifference. Under (16), if B–C settles at time t, peripheral A's posterior about C drops to $\hat{z}_C(t) = z_C(t) \cdot \lambda^{AG}/(\lambda^{AG} + \lambda_C(t))$. The AG atom that C then makes to A is $g_C^A(t) = 1 - \hat{z}_C(t)/z_A(t)$. Substituting into A's indifference condition yields exactly (17), which is satisfied by construction. By symmetry, B's indifference holds as well.

Step 2: Existence and uniqueness of the path. Define $h(t) := z_C(t)/z_A(t)$. Since $z_A(t) = z_0 e^{\lambda^{AG} t}$ is known explicitly, the evolution of z_C reduces to the scalar ODE

$$\dot{h}(t) = h(t)\big(\lambda_C(t) - \lambda^{AG}\big), \tag{19}$$

with terminal condition $h(T) = 1$, where $\lambda_C(t) = \frac{\lambda^{AG}}{2}(-1 + \sqrt{1 + 4h(t)})$ is an explicit locally Lipschitz function of h. Picard–Lindelöf therefore gives a unique local solution near T.

Since $h(T) = 1 < 2$, we have $\lambda_C(T) < \lambda^{AG}$ and hence $\dot{h}(T) < 0$, so h is strictly increasing as we move backward from T. Moreover, h cannot reach 2 in finite backward time: as $h \nearrow 2$, $\lambda_C \nearrow \lambda^{AG}$, so $\dot{h} \to 0$, and the solution slows to a halt before reaching 2. Therefore $h(t) \in (1, 2)$ for all $t \in [0, T)$, the solution extends uniquely to all of $[0, T]$, and in particular $h(0) > 1$. The atom $F_C(0) = 1 - 1/h(0) \in (0, 1)$ is strictly positive and uniquely pinned down.

Step 3: C's optimality. At any t in the support of C's concession distribution, C is indifferent between conceding to both A and B simultaneously and receiving $2(1-\alpha)$, or conceding to only one and receiving $1-\alpha$ from that concession, and then receiving $1-\alpha$ from the other interaction.

Step 4: Payoffs. Since A is indifferent throughout and concedes with positive density from $t = 0$, her payoff equals the value of conceding at $t = 0$: $v_A^* = F_C(0) \cdot \alpha + (1 - F_C(0)) \cdot (1-\alpha) = (1-\alpha) + (2\alpha - 1)F_C(0) > 1 - \alpha$. Player C's payoff is $2(1-\alpha)$ by indifference. □

Remark 1. *Naturally, if $z_C(0) \approx z_0$ but $z_C(0) > z_0$, then also C would concede with an atom in this equilibrium. Thus, C's payoff would be strictly lower than the bilateral AG benchmark. Thus, partial observability of concessions amplifies C's disadvantage in this equilibrium.*

D. The Four-Player Star

This appendix extends the analysis to a four-player star network in which player C bargains simultaneously with three peripheral players 1, 2, and 3. All surplus shares are equal ($\pi_{iC} = 1$ for all i), and all other features of the model are as in Section 2.

D.1. Indifference conditions

Fix the no-concession subgame and suppose all three peripherals and C are active at time t. Let $\lambda_i(t)$ denote the hazard rate of peripheral $i \in \{1,2,3\}$ and $\lambda_C(t)$ that of C. When peripheral j concedes at time t, the continuation game is the three-player star among $\{i,k,C\}$ (where $\{i,k\} = \{1,2,3\} \setminus \{j\}$) with posteriors $(z_i(t), z_k(t), z_C(t))$. We denote by $\mathcal{V}_i^{(j)}(t)$ peripheral i's equilibrium payoff in this three-player continuation and by $\mathcal{W}_C^{(j)}(t)$ the center's equilibrium payoff from the remaining two negotiations in that continuation. These objects are given in closed form by Proposition A.1.

When C concedes, she is revealed rational and concedes in all negotiations simultaneously, yielding α to each peripheral. Each peripheral i's indifference condition is:

$$\lambda_C(t) + \sum_{j \neq i} \lambda_j(t)\, \hat{g}_i^{(j)}(t) = \lambda^{AG}, \tag{20}$$

where

$$\hat{g}_i^{(j)}(t) := \frac{\mathcal{V}_i^{(j)}(t) - (1-\alpha)}{2\alpha - 1}$$

measures the "reputational bonus" peripheral i receives when rival j concedes, relative to i's payoff from conceding herself.

The center's indifference condition is:

$$\sum_{j=1}^{3} \lambda_j(t)\, \Gamma_j(t) = 3\lambda^{AG}, \tag{21}$$

where

$$\Gamma_j(t) := \frac{\alpha + \mathcal{W}_C^{(j)}(t) - 3(1-\alpha)}{2\alpha - 1}$$

measures the center's gain when peripheral j concedes (receiving α from j plus the continuation value from the remaining two negotiations) relative to the center's payoff from conceding on all three negotiations.

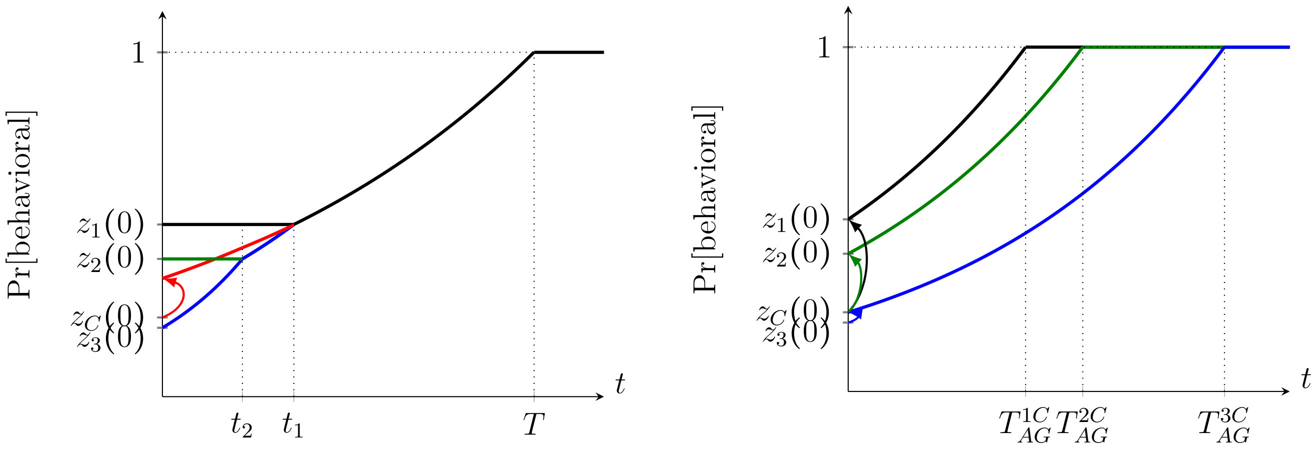
In the three-player model, the analogous continuation values are bilateral AG payoffs, which are simple closed-form functions of posteriors. This yields constant hazard rates in each phase. In the four-player model, $\hat{g}_i^{(j)}(t)$ and $\Gamma_j(t)$ encode three-player equilibrium payoffs that depend on the evolving posterior vector, so (20)–(21) generally produce time-varying hazard rates.

D.2. Equilibrium structure

Despite the added complexity, a numerical example illustrates that the equilibrium retains the sequential activation structure. For the parameterization $z_1(0) = 0.5 > z_2(0) = 0.4 > z_C(0) = 0.23 > z_3(0) = 0.2$ with $r = 1$ and $\alpha = 0.7$, we solve the system (20)–(21) numerically via backward integration from the terminal time T (at which all posteriors reach 1) and obtain a three-phase equilibrium:

- **Phase I** ($t \in [0, t_2]$): Only peripheral 3 and C are active, with constant hazard rates $\lambda_3 = 3\lambda^{AG}$ and $\lambda_C = \lambda^{AG}$. Peripherals 1 and 2 are inactive. C concedes with an atom $F_C(0) \approx 0.33$ at $t = 0$.
- **Phase II** ($t \in [t_2, t_1]$): Peripheral 2 activates. All of $\{2, 3, C\}$ concede with approximately constant hazard rates $\lambda_2 \approx \lambda_3 \approx 1.5\lambda^{AG}$ and $\lambda_C \approx \lambda^{AG}$, while peripheral 1 remains inactive.
- **Phase III** ($t \in [t_1, T]$): All four players concede at λ^{AG} until the common terminal time T.

Figure 4 illustrates the posterior dynamics alongside the bilateral AG benchmark.



(a) Multilateral bargaining

(b) Bilateral bargaining (AG benchmark)

Figure 4: Four-player star: comparison of concession behavior. Multilateral bargaining (left) vs. bilateral bargaining (right). Parameters: $r = 1$, $\alpha = 0.7$, $z_1(0) = 0.5$, $z_2(0) = 0.4$, $z_C(0) = 0.23$, $z_3(0) = 0.2$.

D.3. Payoff comparison

Table 1 compares each player's equilibrium payoff under multilateral bargaining with the bilateral AG benchmark.

	Peripheral 1	Peripheral 2	Peripheral 3	Center C
Multilateral (v_i^*)	0.397	0.449	0.432	0.900
AG benchmark (v_i^{AG})	0.516	0.470	0.300	0.952
Difference	−0.119	−0.021	+0.132	−0.052

Table 1: Equilibrium payoffs: four-player multilateral vs. bilateral AG benchmark.

The payoff pattern extends the three-player results but reveals an additional implication. The weakest peripheral (player 3) is strictly better off under multilateral bargaining, gaining 0.132. The center is strictly worse off, losing 0.052 relative to the sum of her bilateral payoffs.

But now peripheral 2—who is the *intermediate* peripheral—is also worse off, losing 0.021, and the strongest peripheral (player 1) suffers the largest loss of 0.119. This suggests a sharper conclusion than in the three-player case: reputational spillovers benefit only the weakest peripheral, while all other players—including moderately weak peripherals—can be made worse off. The magnitude of the strongest peripheral's loss is substantial, reflecting the extended initial phase during which player 1 remains inactive while C and the weaker peripherals "bargain in earnest."